Ph.D. (Mathematics) Entrance Test

Session 2022-23 & Onward

The test shall consist of 100 marks. 60 marks will be for objective type questions and 40 marks will be for subjective type questions. Each objective type question will have four multiple choice answer having only one correct answer. For each correct answer of objective type question 2 marks will be allotted. Only 30 objective type questions will be set and only 8 questions of five marks and of at most 50 words will be set for each short answer type questions. The medium of entrance examination will be English. No revaluation will be permitted in any circumstances.

Course content

Analysis: Elementary set theory, finite, countable and uncountable sets, Real number system as a complete ordered field, Archimedean property, supremum, infimum. Sequences and series, convergence, limsup, liminf. Bolzano Weierstrass theorem, Heine Borel theorem. Continuity, uniform continuity, differentiability, mean value theorem. Sequences and series of functions, uniform convergence. Riemann sums and Riemann integral, Improper Integrals. Monotonic functions, types of discontinuity, functions of bounded variation, Riemann-Stieltjes integral. Metric spaces, compactness, connectedness.

Linear Algebra: Vector spaces, subspaces, linear dependence, basis, dimension, algebra of linear transformations. Algebra of matrices, rank and determinant of matrices, linear equations. Eigenvalues and eigenvectors, Cayley-Hamilton theorem. Matrix representation of linear transformations. Change of basis, canonical forms, diagonal forms, triangular forms, Jordan forms. Inner product spaces, orthonormal basis. Quadratic forms, reduction and classification of quadratic forms

Complex Analysis: Algebra of complex numbers, the complex plane, polynomials, power series, transcendental functions such as exponential, trigonometric and hyperbolic functions. Analytic functions, Cauchy-Riemann equations. Contour integral, Cauchy's theorem, Cauchy's integral formula, Liouville's theorem, Maximum modulus principle, Schwarz lemma, Open mapping theorem. Taylor series, Laurent series, calculus of residues. Conformal mappings, Mobius transformations.

Algebra: Groups, subgroups, normal subgroups, quotient groups, homomorphisms, cyclic groups, permutation groups, Cayley's theorem, class equations, Sylow theorems. Rings, ideals, prime and maximal ideals, quotient rings, unique factorization domain, principal ideal domain, Euclidean domain. Polynomial rings and irreducibility criteria. Fields, finite fields, field extensions, Galois Theory.

Ordinary Differential Equations (ODEs):

Existence and uniqueness of solutions of initial value problems for first order ordinary differential equations, singular solutions of first order ODEs, system of first order ODEs. General theory of homogenous and non-homogeneous linear ODEs, variation of parameters, Sturm-Liouville boundary value problem, Green's function.

Partial Differential Equations (PDEs):

Lagrange and Charpit methods for solving first order PDEs, Cauchy problem for first order PDEs. Classification of second order PDEs, General solution of higher order PDEs with constant coefficients, Method of separation of variables for Laplace, Heat and Wave equations.

Topology: basis, dense sets, subspace and product topology, separation axioms, connectedness and compactness.

Functional Analysis: Normed linear spaces. Banach spaces and examples. Quotient space of normed linear spaces and its completeness, equivalent norms. Riesz Lemma, basic properties of finite dimensional normed linear spaces and compactness. Weak convergence and bounded linear transformations, normed linear spaces of bounded linear transformations, dual spaces with examples. Inner product spaces. Hilbert spaces. Orthonormal Sets. Bessel's inequality. Complete orthonormal sets and Parseval's identity. Structure of Hilbert spaces. Projection theorem. Riesz representation theorem. Adjoint of an operator on a Hilbert space. Reflexivity of Hilbert spaces.

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