Centre for Basic Sciences (CBS)

COURSE STRUCTURE

SCHEME OF EXAMINATION

&

SYLLABUS

of

Integrated M.Sc. Mathematics

UNDER

FACULTY OF SCIENCE

Approved by Board of Studies in Mathematics

Effective from July 2022 onward

Center for Basic Sciences,
Pt Ravishankar Shukla University,
Raipur, Chhattisgarh - 492010
Contact No. – 0771-2262216

Contents

7/10/22

Title of the Program: Integrated M.Sc. Mathematics

Program Objective

To impart fundamental and computational knowledge of mathematics to students to develop world-class academician, researcher and mathematics teachers who can understand their responsibilities in solving social and ethical issues with a scientific approach for the betterment of society.

Program Outcomes

PO1: To have fundamental as well as applied knowledge of the various fields of Mathematics.

PO2: To develop good practical knowledge of computational techniques using various programming languages and their applications.

PO3: To orient the students to be able to work in research organizations of national and international repute and become the top future scientists of the country.

PO4: To promote the students to explore and foster connections with other fields for interdisciplinary knowledge.

PO5: To develop world-class mathematics teachers who can understand their responsibilities in solving social and ethical issues with a scientific approach for the betterment of society.

Program Specific Outcomes

PSO1: Students will have knowledge of fundamental as well as applied aspects of various fields of Mathematics along with the foundation of Physics, Chemistry, Biology and Programming Languages.

PSO2: Students will have the knowledge of Mathematical Foundations, Algebra, Analysis, Topology, Geometry, Statistics, Probability Theory, Stochastic Process, Discrete Mathematics, Mathematical Modeling, Mathematical Biology along with Numerical Techniques and Mathematical Computing with applications in various areas.

PSO3: Students will develop skills for interdisciplinary research, critical thinking and problem-solving ability.

PSO4: Students will be able to not only design models for real-life problems but also analyze and interpret the model independently.

PSO5: Activities like reading project, review writing, presentations will inculcate the abilities of better written as well as oral expression of the scientific work.

General Pattern of the Program

Courses offered during the first year (Semesters I to II) are meant as basic and introductory courses in Biology, Chemistry, Mathematics, Physics and Environmental Science. These are common and mandatory for all students. These courses are intended to give a flavor of the various approaches and analyses and to prepare the students for advanced courses in later years of study. In addition, there will be Interdisciplinary Courses for computational skills using mathematical methods. Students are also given training to develop skills in Communication, Creative Hindi & Scientific Writing and History of Science through courses in Humanities. In the second year (Semester - III), students have the freedom to choose their stream (Biology, Chemistry, Mathematics, Physics) for masters program on the bases of their interest. Courses offered in the first two years would help them make an informed judgment to determine their real interest and aptitude for a given subject. One of the important features that the CBS has adopted is semester-long projects called Lab Training / Theory projects, which are given the same weightage as a regular course. By availing this, a student can work in an experimental lab or take up a theory project every semester. This is meant to help the student get trained in research methodology, which will form a good basis for the 9$^{\text{th}}$ semester project work in the fifth year. The subjects/courses are described further with their credit points. Few courses are common to different streams.

Course Structure of
Integrated M.Sc. (Mathematics)

Effective from July, 2022

- Minimum total credits for integrated M.Sc. degree is 240.
- Semesters I to VIII will carry 25 credits each.
- Semesters IX and X will carry 20 credits each.

Abbreviation: B: Biology, C: Chemistry, M: Mathematics, P: Physics, G: General, H: Humanities, BL: Biology Laboratory, CL: Chemistry Laboratory, PL: Physics Laboratory, GL: General Laboratory, ML: Mathematics Laboratory; ME: Mathematics Elective, MPr: Mathematics Project

First Year

Integrated M.Sc., Semester - I

Subject Code	Subject	Contact Hours/ Week [Theory + Tutorial]	Credit
B101	Biology - I	[2+1]	3
C101	Chemistry - I	[2+1]	3
M101/MB101	Mathematics - I	[2+1]	3
P101	Physics- I	[2+1]	3
G101	Computer Basics	[2+1]	3
H101	Communication Skills	[2+0]	2
		Contact Hours/ Week Laboratory	
BL101	Biology Laboratory-I	[4]	2
CL101	Chemistry Laboratory-I	[4]	2
PL101	Physics Laboratory-I	[4]	2
GL101	Computer Laboratory	[4]	2
	(25 of 240 credits)	**Total:**	25
Additional Papers		**Contact Hours/ Week [Theory + Tutorial]**	
ES101	Environmental Studies	[2+0]	2

Integrated M.Sc., Semester - II

Subject Code	Subject	Contact Hours/ Week [Theory + Tutorial]	Credit
B201	Biology - II	[2+1]	3
C201	Chemistry - II	[2+1]	3
M201/MB201	Mathematics - II	[2+1]	3
P201	Physics- II	[2+1]	3
G201	Electronics and Instrumentation	[2+1]	3
		Contact Hours/ Week Laboratory	
BL201	Biology Laboratory-II	[4]	2
CL201	Chemistry Laboratory-II	[4]	2
PL201	Physics Laboratory-II	[4]	2
GL201	Electronics Laboratory	[4]	2
H201	Communication Skills Lab	[4]	2
	(50 of 240 credits)	**Total:**	25
Additional Papers			
ES201	Environmental Studies	[2]	2

Second Year

Integrated M.Sc. Mathematics, Semester - III

Subject Code	Subject	Contact Hours/ Week [Theory + Tutorial]	Credit
M301	Mathematical Foundations	[3+1]	4
M302	Analysis - I	[3+1]	4
M303	Algebra - I	[3+1]	4
M304	Elementary Number Theory	[3+1]	4
M305	Computational Mathematics-I	[3+1]	4
H301	Creative Hindi	[2+0]	2
H302	History and Philosophy of Science	[2+0]	2
		Contact Hours/ Week Laboratory	
GL301	Computational Mathematics Laboratory-I	[2]	1
	(75 of 240 credits)	**Total:**	25

Integrated M.Sc. Mathematics, Semester - IV

Subject Code	Subject	Contact Hours/ Week [Theory + Tutorial]	Credit
M401	Analysis-II	[3+1]	4
M402	Algebra - II	[3+1]	4
M403	Introduction to Differential Equations	[3+1]	4
M404	Topology-I	[3+1]	4
G401	Statistical Techniques and Applications	[3+1]	4
		Contact Hours/ Week Laboratory	
GL401	Computational Laboratory and Numerical Methods	[4]	2
GL402	Statistical Techniques Laboratory	[2]	1
H401	Communication Skills Lab-II	[4]	2
	(100 of 240 credits)	**Total:**	25

Third Year

Integrated M.Sc. Mathematics, Semester - V

Subject Code	Subject	Contact Hours/ Week [Theory + Tutorial]	Credit
M501	Analysis-III	[3+1]	4
M502	Algebra - III	[3+1]	4
M503	Topology - II	[3+1]	4
M504	Probability Theory	[3+1]	4
PM501	Numerical Analysis	[3+1]	4
H501	Scientific Writing in Hindi	[2+0]	2
		Contact Hours/ Week Laboratory	
PML501	Numerical Methods Laboratory	[6]	3
	(125 of 240 credits)	**Total:**	25

Integrated M.Sc. Mathematics, Semester - VI

Subject Code	Subject	Contact Hours/ Week [Theory + Tutorial]	Credit
M601	Analysis-IV	[3+1]	4
M602	Algebra - IV	[3+1]	4
M603	Partial Differential Equations	[3+1]	4
M604	Ordinary Differential Equations	[3+1]	4
M605	Numerical Analysis of Partial Differential Equations	[3+1]	4
H601	Ethics of Science and IPR	[2]	2
H602	Scientific Writing in English	[2]	2
		Contact Hours/ Week Laboratory	
ML601	Computational Mathematics Laboratory-III	[2]	1
	(150 of 240 credits)	**Total:**	25

Fourth Year

Integrated M.Sc. Mathematics, Semester - VII

Subject Code	Subject	Contact Hours/ Week [Theory + Tutorial]	Credit
M701	Functional Analysis	[3+1]	4
M702	Discrete Mathematics	[3+1]	4
M703	Introduction to Mathematical Modelling	[3+1]	4
M704	Operations Research	[3+1]	4
M705	Stochastic Analysis	[3+1]	4
Project		Contact Hours/ Week	
MPr701	Reading Project	[10]	5
	(175 of 240 credits)	**Total:**	25

Integrated M.Sc. Mathematics, Semester - VIII

Subject Code	Subject	Contact Hours/ Week [Theory + Tutorial]	Credit
M801	Graph Theory	[3+1]	4
M802	Advanced Discrete Mathematics	[3+1]	4
M803	Nonlinear Dynamics and Chaos	[3+1]	4
M804	Mathematical Biology	[3+1]	4
M805	Computational Mathematics III	[3+1]	4
Project		Contact Hours/ Week	
MPr801	Project	[10]	5
	(200 of 240 credits)	**Total:**	25

Fifth Year

Integrated M.Sc. Mathematics, Semester - IX

Subject Code	Subject	Contact Hours/ Week [Theory + Tutorial]	Credit
MPr901	Project		20
	(220 of 240 credits)	Total:	20

Integrated M.Sc. Mathematics, Semester - X

Subject Code*	Subject	Contact Hours/ Week [Theory + Tutorial]	Credit
ME1001	Elective-I	[4+1]	5
ME1002	Elective-II	[4+1]	5
ME1003	Elective-III	[4+1]	5
ME1004	Elective-IV	[4+1]	5
	(240 of 240 credits)	Total:	20

*Four subjects will be offered according to the availability of the instructors and minimum number of students taking a course. The chosen four subject will have subjects codes ME1001, ME1002, ME1003 and ME1004.

Integrated M.Sc. Mathematics, Semester - X: Electives

Elective No	Subject
ME01	Dynamical Systems Using Matlab
ME02	Commutative Algebra
ME03	Financial Mathematics
ME04	Nonlinear Analysis
ME05	Differential Topology
ME06	Introduction to Cryptography
ME07	Introduction to Nonlinear Optimization
ME08	Complex Network
ME09	Representation Theory of Finite Groups
ME10	Algebraic Number Theory
ME11	Algebraic Topology
ME12	Differential Geometry & Applications
ME13	Fuzzy Set Theory & Its Applications
ME14	Wavelets
ME15	Mathematical Methods
ME16	Fourier Analysis

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Scheme of Examination
Integrated M.Sc. in Mathematics

First Year

Integrated M.Sc. Semester - I

Subject Code	Subject	Internal Marks		External Marks		Total Marks	Credit
		Max	Min	Max	Min	Max	
B101	Biology - I	60	24	40	16	100	3
C101	Chemistry - I	60	24	40	16	100	3
M101/MB101	Mathematics - I	60	24	40	16	100	3
P101	Introductory Physics- I	60	24	40	16	100	3
G101	Computer Basics	60	24	40	16	100	3
H101	Communication Skills	60	24	40	16	100	2
Practical							
BL101	Biology Laboratory-I	60	24	40	16	100	2
CL101	Chemistry Laboratory-I	60	24	40	16	100	2
PL101	Physics Laboratory-I	60	24	40	16	100	2
GL101	Computer Laboratory	60	24	40	16	100	2
Additional Papers							
ES101	Environmental Studies	60	24	40	16	100	2

Integrated M.Sc. Semester - II

Subject Code	Subject	Internal Marks		External Marks		Total Marks	Credit
		Max	Min	Max	Min	Max	
B201	Biology - II	60	24	40	16	100	3
C201	Chemistry - II	60	24	40	16	100	3
M201/MB201	Mathematics - II	60	24	40	16	100	3
P201	Physics- II	60	24	40	16	100	3
G201	Electronics and Instrumentation	60	24	40	16	100	3
Practical							
BL201	Biology Laboratory-II	60	24	40	16	100	2
CL201	Chemistry Laboratory-II	60	24	40	16	100	2
PL201	Physics Laboratory-II	60	24	40	16	100	2
GL201	Electronics Laboratory	60	24	40	16	100	2
H201	Communication Skills Lab	60	24	40	16	100	2
Additional Papers							
ES201	Environmental Studies	60	24	40	16	100	2

Second Year

Integrated M.Sc. Mathematics, Semester - III

Subject Code	Subject	Internal Marks		External Marks		Total Marks	Credit
		Max	Min	Max	Min	Max	
M301	Mathematical Foundations	60	24	40	16	100	4
M302	Analysis - I	60	24	40	16	100	4
M303	Algebra - I	60	24	40	16	100	4
M304	Elementary Number Theory	60	24	40	16	100	4
M305	Computational Mathematics-I	60	24	40	16	100	4
H301	Creative Hindi	60	24	40	16	100	2
H302	History and Philosophy of Science	60	24	40	16	100	2
Practical							
GL301	Computational Mathematics Laboratory-I	60	24	40	16	100	1

Integrated M.Sc. Mathematics, Semester - IV

Subject Code	Subject	Internal Marks		External Marks		Total Marks	Credit
		Max	Min	Max	Min	Max	
M401	Analysis-II	60	24	40	16	100	4
M402	Algebra - II	60	24	40	16	100	4
M403	Introduction to Differential Equations	60	24	40	16	100	4
M404	Topology-I	60	24	40	16	100	4
G401	Statistical Techniques and Applications	60	24	40	16	100	4
Practical							
GL401	Computational Laboratory and Numerical Methods	60	24	40	16	100	2
GL401	Statistical Techniques Laboratory	60	24	40	16	100	1
H401	Communication Skills Lab-II	60	24	40	16	100	2

Third Year

Integrated M.Sc. Mathematics, Semester - V

Subject Code	Subject	Internal Marks		External Marks		Total Marks	Credit
		Max	Min	Max	Min	Max	
M501	Analysis-III	60	24	40	16	100	4
M502	Algebra - III	60	24	40	16	100	4
M503	Topology - II	60	24	40	16	100	4
M504	Probability Theory	60	24	40	16	100	4
PM501	Numerical Analysis	60	24	40	16	100	4
H501	Scientific Writing in Hindi	60	24	40	16	100	2
Practical							
PML501	Numerical Methods Laboratory	60	24	40	16	100	3

Integrated M.Sc. Mathematics, Semester - VI

Subject Code	Subject	Internal Marks		External Marks		Total Marks	Credit
		Max	Min	Max	Min	Max	
M601	Analysis-IV	60	24	40	16	100	5
M602	Algebra - IV	60	24	40	16	100	5
M603	Partial Differential Equations	60	24	40	16	100	4
M604	Ordinary Differential Equations	60	24	40	16	100	4
M605	Numerical Analysis of Partial Differential Equations	60	24	40	16	100	4
H601	Ethics of Science and IPR	60	24	40	16	100	2
H602	Scientific Writing in English	60	24	40	16	100	2
Practical							
ML601	Computational Mathematics Laboratory-III	60	24	40	16	100	3

Fourth Year

Integrated M.Sc. Mathematics, Semester - VII

Subject Code	Subject	Internal Marks		External Marks		Total Marks	Credit
		Max	Min	Max	Min	Max	
M701	Functional Analysis	60	24	40	16	100	4
M702	Discrete Mathematics	60	24	40	16	100	4
M703	Introduction to Mathematical Modelling	60	24	40	16	100	4
M704	Operations Research	60	24	40	16	100	4
M705	Stochastic Analysis	60	24	40	16	100	4
Project							
MPr701	Reading Project	60	24	40	16	100	5

Integrated M.Sc. Mathematics, Semester - VIII

Subject Code	Subject	Internal Marks		External Marks		Total Marks	Credit
		Max	Min	Max	Min	Max	
M801	Graph Theory	60	24	40	16	100	4
M802	Advanced Discrete Mathematics	60	24	40	16	100	4
M803	Nonlinear Dynamics and Chaos	60	24	40	16	100	4
M804	Mathematical Biology	60	24	40	16	100	4
M805	Computational Mathematics III	60	24	40	16	100	4
Project							
MPr801	Project	60	24	40	16	100	5

Fifth Year

Integrated M.Sc. Mathematics, Semester - IX

Subject Code	Subject	Project Report/ Dissertation		Seminar Based on Project		Viva-Voce Based on Project Report and Seminar		Total Marks Max	Credit
		Max	Min	Max	Min	Max	Min		
MPr901	Project	150	60	150	60	100	40	400	20

Integrated M.Sc. Mathematics, Semester - X

Subject Code*	Subject	Internal Marks		External Marks		Total Marks	Credit
		Max	Min	Max	Min	Max	
ME1001	Elective-I	60	24	40	16	100	5
ME1002	Elective-II	60	24	40	16	100	5
ME1003	Elective-III	60	24	40	16	100	5
ME1004	Elective-IV	60	24	40	16	100	5

*Elective subjects will be offered according to the availability of instructors and minimum number of interested students taking a course from the list of elective subjects in the syllabus. The chosen four subjects will have codes ME1001, ME1002, ME1003 and ME1004.

Syllabus of Integrated M.Sc. (Mathematics)

1 Semester-I

1.1 M101: Mathematics-I

Learning Outcomes: At the end of the course, the students will be able to :

1. Understand the mathematical concept of number systems, their algebraic and order properties.
2. Learn the techniques of mathematical proof and mathematical writing.
3. Ability to manipulate function and relation for solving mathematical problems.
4. Understand the concept of limit of a functions and its properties.
5. Understand the concept of sequence and series of real numbers and their properties.
6. Analyse convergence of sequence and series of real numbers.

Contents:

Unit-I Introduction of Number Systems: Natural Numbers, Algebraic Properties, Mathematical Induction. Real Numbers, Order Properties and Completeness Property of $\mathbb{R}$, Intervals on $\mathbb{R}$, Infinity, Infinite Sets and Cardinality.

Unit-II Reading and Writing Mathematics: Illustration of mathematical proofs via examples, Illustration of Conjunction, Disjunction, Negation of Statements and Conditional Statements via examples. Techniques of mathematical proofs.

Unit-III Functions and Relations: Sets, De Morgan's Laws, Relations, Cartesian Products, Functions and Graphical Representation, Injective and Surjective functions, Composition and Inverse of Functions, Level Sets, Equivalence Relations and Equivalence Classes.
Limits: Limits of Functions, Boundedness, Squeeze Theorem, Limits at Infinity.

Unit-IV Sequences: Sequences, Convergence, Limit theorems, Divergence, Cauchy Sequences.

Unit-V Infinite Series: Convergence and Divergence of Series, Geometric Series, Tests for Convergence.

References:

[1] Subhash Chandra Malik and Savita Arora. *Mathematical analysis.* New Age International, 2012.

[2] Ajit Kumar, S Kumaresan, and Bhaba Kumar Sarma. *A Foundation Course in Mathematics.* Alpha Science International Limited, 2018.

[3] Donald R Sherbert and Robert G Bartle. *An Introduction to Real Analysis.* John Wiley & Sons, Inc., 2014.

[4] D'Angelo John Philip and Douglas Brent West. *Mathematical thinking: problem-solving and proofs.* Prentice-Hall, 1997.

[5] James R Munkres. *Topology.* Prentice Hall, 2000.

1.2 MB101: Remedial Mathematics-I

Learning Outcomes: At the end of the course, the students will be able to :

1. Understand and apply the concept of trigonometry and vectors.
2. Understand the concept of different number systems and their properties.
3. Analyse AP, GP and HP series and their applications.
4. Understand the concept of limit, continuity, differentiability of real functions.
5. Explore properties and applications of continuity and differentiability.

Contents:

Unit-I Trigonometry and Vectors: Polar coordinates, relations between different trigonometric functions, periodicity, graphical representation, fundamental identities, addition formulae, multiple angles, factorization formulae. Scalers and vectors, norm of a vector, dot product, projections, cross product.
Sets and Functions: Sets, Functions, Inequalities, graphical representation.

Unit-II Numbers: Numbers of Different Types ($\mathbf{N}, \mathbf{Z}, \mathbf{R}, \mathbf{R}\backslash\mathbf{Q}$), Algebraic Properties, Factorial notation, Mathematical Induction, Division Algorithm, Divisibility, Prime Numbers, Fundamental Theorem of Arithmetic, Order Properties and Completeness Property of $\mathbf{R}$, concept of congruences.

Unit-III Series: AP, GP and HP and inequalities of the mean, Sum of a series, Sigma notation, Convergence, Limit Theorems, Divergence Tests for Convergence (Absolute Convergence and Non-absolute Convergence), Series of Functions, Taylor's Series, Power Series.

Unit-IV Limits and Continuity: Limits of functions, Boundedness, Squeeze Theorem. Graphical idea of monotonic function and Continuity, Continuous Functions, Continuous Functions on Intervals, Uniform Continuity.

Unit-V Derivatives and Differentiation: Definition and Graphical Representation of Derivatives, Differentiability and Continuity, Chain Rule, Product and quotient rules, Higher Derivatives. Derivatives of Exponential, Logarithmic, Trigonometric and Inverse Trigonometric functions, derivatives of inverse functions, derivatives of Power Series. Mean Value Theorem, Derivatives and Extrema, L'Hospital's Rule.

References:

[1] Ajit Kumar, S Kumaresan, and Bhaba Kumar Sarma. *A Foundation Course in Mathematics.* Alpha Science International Limited, 2018.

[2] Donald R Sherbert and Robert G Bartle. *An Introduction to Real Analysis.* John Wiley & Sons, Inc., 2014.

[3] Maurice D Weir, George Brinton Thomas, Joel Hass, and Frank R Giordano. *Thomas' calculus.* Pearson Education, 2018.

[4] James Stewart. *Single variable calculus: Concepts and contexts.* Cengage Learning, 2018.

[5] Gilbert Strang and Edwin Herman. *Calculus.* OpenStax Houston, Texas, 2016.

[6] TM Apostol. *Mathematical Analysis.* Pearson Education, Inc, 2004.

1.3 G101: Computer Basics (Programming in C)

Learning Outcomes: At the end of the course, the students will be able to :

1. Understand concept of the structured programming language.
2. Understand syntax, data representation, input, out, data types, compilation.
3. Apply control structures to solve problems.
4. Apply Structure and Union data types to solve programming problems.
5. Understand concept of pointers and its usage.

Contents:

Unit-I Introduction to C programming structure and C compiler. Data representation: Simple data types like real integer, character etc. Program, statements and Header Files. Simple Input Output statements in C. Running simple C programs, Data Types. Operators and Expressions.

Unit-II Control Structure: If statement, If-else statement. Compound Statement. Loops: For - loop. While - loop. Do-While loop, Break and exit statements, Switch statement, Continue statement, Goto statement.

Unit-III Array. Types of Array, String Handling. Functions: Function main, Functions accepting more than one parameter, User defined and library functions. Concept associatively with functions, function parameter, Return value, recursion function.

Unit-IV Structure and Union, Declaring and using Structure, Structure initialization, Structure within Structure, Operations on Structures, Array of Structure. Array within Structure, Structure and Functions, Union, Scope of Union, Difference between Structure and Union.

Unit-V Pointers Definition and use of pointer, address operator, pointer variable, referencing pointer, void pointers, pointer arithmetic, pointer to pointer, pointer and arrays, passing arrays to functions, pointer and functions, accessing array inside functions. pointers and two dimensional arrays, array of pointers. pointers constants, pointer and strings.

References:

[1] Venu Gopal. *Mastering C.* McGraw-Hill Education (India) Pvt Limited, 2006.

[2] V Rajaraman and Neeharika Adabala. *Fundamentals of computers.* PHI Learning Pvt. Ltd., 2014.

[3] Yashvant Kanethkar. *Let Us, C.* BPB publications, 2018.

1.4 GL101: Programming in C

Learning Outcomes: At the end of the course, the students will be able to :

1. Understand syntax, data representation, input, out, data types, compilation and apply them into programs.
2. Apply control structures to solve problems.
3. Apply Structure and Union data types to solve programming problems.
4. Apply concept of pointers to make program to solve problems.

Contents: Practicals of programming in C are based on syllabus of G-101.

References:

[1] Venu Gopal. *Mastering C.* McGraw-Hill Education (India) Pvt Limited, 2006.

[2] V Rajaraman and Neeharika Adabala. *Fundamentals of computers.* PHI Learning Pvt. Ltd., 2014.

[3] Yashvant Kanethkar. *Let Us, C.* BPB publications, 2018.

2 Semester-II

2.1 M201: Mathematics-II

Learning Outcomes: At the end of the course, the students will be able to :

1. Understand concept of continuity, properties of continuous functions.
2. Explore the concept of Derivative and related properties.
3. Understand application of derivatives, e.g., maxima/minima, convexity etc.
4. Understand the concept of Reimann integrations, its properties.
5. Able to apply integration theory to calculate arc length, area of surface, volume of a solid of revolution etc.

Contents:

Unit-I Continuity: Continuous Functions, Graphical Representation, Composition and Inverse of Continuous Functions, Continuous Functions on Intervals.
Differentiation: Definition and Graphical Representation of Derivatives, Differentiability and Continuity, Chain Rule, Higher Derivatives. Mean Value Theorems, Derivatives and Extrema, L'Hospital's Rule, Taylor's Theorem and Applications.

Unit-II Maxima and Minima: Sufficient conditions for a function to be increasing/decreasing, Sufficient conditions for a local extremum, Absolute minimum/maximum, Convex/concave functions.

Unit-III Integration: Riemann Integral and its Properties, Statement of Fundamental Theorem of Calculus. Applications of Integration: Arc length of a plane curve, Arc length of a plane curve in parametric form, Area of a surface of revolution, Volume of a solid of revolution by slicing, by the washer method and by the shell method.

Unit - IV Limit and Continuity of Scalar Fields: Spaces $\mathbb{R}^2$ and $\mathbb{R}^3$, Scalar fields, level curves and contour lines, Limit of a scalar field, Continuity of a scalar field, Properties of continuous scalar fields. Differentiation of Scalar Fields: Partial derivatives, Differentiability, Chain rules, Implicit differentiation, Directional derivatives, Gradient of a scalar field, Tangent plane and normal to a surface, Higher order partial derivatives, Maxima and minima, Saddle points, Second derivative test for maxima/minima/saddle points.

Unit-V Complex Numbers: Complex Numbers, Statement of Fundamental Theorem of Algebra, Polar Coordinates, Euler's and de Moivre's Formulae, Formulae for Sine and Cosine, Powers and roots of complex numbers, The exponential and trigonometric functions, Hyperbolic functions, Logarithms, Complex roots and powers, Inverse trigonometric and hyperbolic functions.

References:

[1] Mary L Boas. *Mathematical methods in the physical sciences.* John Wiley & Sons, 2006.

[2] Peter D Lax and Maria Shea Terrell. *Calculus with applications.* Springer, 2020.

[3] Kenneth A Ross. *Elementary Analysis.* Springer, 2013.

[4] Maurice D Weir, George Brinton Thomas, Joel Hass, and Frank R Giordano. *Thomas' calculus.* Pearson Education, 2018.

[5] James Stewart. *Single variable calculus: Concepts and contexts.* Cengage Learning, 2018.

[6] Gilbert Strang and Edwin Herman. *Calculus.* OpenStax Houston, Texas, 2016.

2.2 MB201: Remedial Mathematics-II

Learning Outcomes: At the end of the course, the students will be able to :

1. Understand the concept of integration and its properties.
2. Comprehend relation between integration and differentiation, application of integration.
3. Understand complex number systems and their properties.
4. Able to solve system of linear equations by matrix methods.
5. Explore the basic notion of combinatorics and probability, their properties and applications.
6. Explore basic statistics, its uses, scope and limitations.

Contents:

Unit-I Integration: Notion of an integral, integral as limit of sums, anti-derivatives, area under a curve, Fundamental theorem of calculus, definite integrals, indefinite integrals, Rules of integration: integration by parts, integration by substitution, Properties of definite integrals, Application of integrals (path lengths, areas, volumes, etc.).

Unit-II Complex Numbers: real and imaginary parts, the complex plane, complex algebra (complex conjugate, absolute value, complex equations, graphs, physical applications). Consequences of Euler's formula.

Unit-III Matrices and Linear Equations: System of linear equations, notion of a matrix, determinant. Row and column operations, Gauss Elimination, Simple properties of matrices and their inverses.

Unit-IV Combinatorics and Probability: Permutations and combinations, Binomial theorem for integral and non-integral powers, Pascal's triangle, Introductory probability theory, Conditional probability, Binomial probability distribution.

Unit-V Basic Statistics: frequency tables, measure of central tendencies (mean, median, mode), measure of variation (standard deviation etc).

References:

[1] TM Apostol. *Mathematical Analysis.* Pearson Education, Inc, 2004.

[2] Saminathan Ponnusamy. *Foundations of mathematical analysis.* Springer Science & Business Media, 2011.

[3] Roger J Barlow. *Statistics: a guide to the use of statistical methods in the physical sciences.* John Wiley & Sons, 1993.

[4] SC Gupta and VK Kapoor. *Fundamentals of mathematical statistics.* Sultan Chand & Sons, 2020.

3 Semester-III

3.1 M301: Mathematical Foundation

Learning Outcomes: At the end of the course, the students will be able to :

1. Understand the principles and practices of their field of study.
2. Understand the difference between necessary and sufficient conditions.

3. Understand Sets, its basic operations.
4. Understand and use the relations equivalence relation and partial order relation.
5. Understand function and able to analyse injective, surjective or bijective functions.
6. Understand the concept of order and order properties on number systems.

Contents:

Unit-I: Logic: Quantifiers, negations, examples of various mathematical and non-mathematical statements. Exercises and examples.
Set Theory: Definitions, subsets, unions, intersections, complements, symmetric difference, De-Morgan's laws for arbitrary collection of sets. Power set of a set.

Unit-II Relations and maps: Cartesian product of two sets. Relations between two sets. Examples of relations. Definition of a map, injective, surjective and bijective maps. A map is invertible if and only if it is bijective. Inverse image of a set with respect to a map. Relation between inverse images and set theoretic operations. Equivalence relations (with lots of examples). Schroeder- Bernstein theorem.

Unit-III Finite and Infinite sets: Finite sets, maps between finite sets, proof that number of elements in a finite set is well defined. Definition of a countable set (inclusive of a finite set). Countably infinite and uncountable sets. Examples. Proof that every infinite set has a proper, countably infinite subset. Uncountability of P(N).

Unit-IV Partially Ordered Sets: Concept of partial order, total order, examples. Chains, Zorn's Lemma.

Unit-V Peano's Axioms. Well-Ordering Principle. Weak and Strong Principles of Mathematical Induction. Transfinite Induction. Axiom of Choice, product of an arbitrary family of sets. Equivalence of Axiom of Choice, Zorn's Lemma and Well-ordering principle.

References:

[1] Ajit Kumar, S Kumaresan, and Bhaba Kumar Sarma. *A Foundation Course in Mathematics.* Alpha Science International Limited, 2018.

[2] Péter Komjáth and Vilmos Totik. *Problems and theorems in classical set theory.* Springer Science & Business Media, 2006.

[3] Stephen Abbott. *Understanding analysis.* Springer, 2001.

[4] Daniel J Velleman. *How to prove it: A structured approach.* Cambridge University Press, 2019.

[5] Daniel Cunningham. *A logical introduction to proof.* Springer Science & Business Media, 2012.

[6] Daniel W Cunningham. *Set Theory: A First Course.* Cambridge University Press, 2016.

[7] R. Lal. *Algebra 1: Groups, Rings, Fields and Arithmetic.* Infosys Science Foundation Series. Springer Singapore, 2017.

3.2 M302: Analysis I

Learning Outcomes: At the end of the course, the students will be able to :

1. Understand the fundamentals of real number systems, algebraic and order properties, ordered field.
2. Analysis convergence of sequence, subsequence, cauchy sequence.
3. Understand convergence of infinite series, cauchy product, power series and radius of convergence.
4. Understand the concepts from analysis of a single real variable (convergence, uniform continuity) in the context of metric spaces.

5. Understand the differentiability and their properties.

6. Understand the concepts of Rolle's theorem, mean value theorems, Taylor's theorem.

Contents:

Unit-I Real Number System: Real number system: Construction via Cauchy sequences. Concept of a field, ordered field, examples of ordered fields, supremum, infimum. Order completeness of **R**, **Q** is not order complete. Absolute values, Archimedean property of **R**. **C** as a field, and the fact that **C** cannot be made into an ordered field. Denseness of **Q** in **R**. Every positive real number has a unique positive n^{th} root.

Unit-II Sequences: Sequences, limit of a sequence, basic properties. Bounded sequences, monotone sequences, convergence of a monotone sequence. Sandwich theorem and its applications. Cauchy's first limit theorem, Cauchy's second limit theorem.

Subsequences and Cauchy sequences: Every sequence of real numbers has a monotone subsequence. Definition of a Cauchy sequence. Cauchy completeness of $\mathbb{R}$, $\mathbb{Q}$ is not Cauchy complete.

Unit-III Infinite Series: Basic notions on the convergence of infinite series. Absolute and conditional convergence. Comparison test, ratio test, root test, alternating series test, Dirichlet's test, Statement of Riemann's rearrangement theorem, Cauchy product of two series. Power series, radius of convergence.

Unit-IV Continuous functions: Continuity, sequential and neighbourhood definitions, basic properties such as sums and products of continuous functions are continuous. Intermediate Value Theorem, Continuous functions on closed and bounded intervals, Monotone continuous functions, inverse functions, Uniform Continuity, examples and counter-examples.

Unit V Differentiable functions: Definition: as a function infinitesimally approximal by a linear map, equivalence with Newton's ratio definition, basic properties. One-sided derivatives, The O; o and ñotations with illustrative examples. Chain rule with complete proof (using above definition). Local monotonicity, relation between the sign of f' and local monotonicity. Proofs of Rolle's theorem and the Cauchy-Lagrange Mean value theorem. L'Hospital's rule and applications. Higher derivatives and Taylor's theorem, estimation of the remainder in Taylor's theorem, Convex functions.

References:

[1] Ajit Kumar and Somaskandan Kumaresan. *A basic course in real analysis.* CRC press, 2014.

[2] Stephen Abbott. *Understanding analysis.* Springer, 2001.

[3] Terence Tao. Analysis ii, texts and readings in mathematics, 2015.

[4] T. Tao. *Analysis I: Third Edition.* Texts and Readings in Mathematics. Springer Singapore, 2016.

[5] W.R. Wade. *Introduction to Analysis: Pearson New International Edition.* Pearson Education, Limited, 2013.

[6] Saminathan Ponnusamy. *Foundations of mathematical analysis.* Springer Science & Business Media, 2011.

[7] Steven G Krantz. *A guide to real variables.* American Mathematical Soc., 2014.

[8] Miklós Laczkovich and Vera T Sós. *Real Analysis: Foundations and Functions of One Variable.* Springer, 2015.

[9] Sadhan Kumar Mapa. *Introduction to Real Analysis.* Sarat Book Distributors, 2014.

3.3 M303 : Algebra - I

Learning Outcomes: At the end of the course, the students will be able to :

1. Understand the concept of groups, homomorphisms, (normal) subgroups, quotient groups and related objects.
2. Understand more advanced results concerning groups and group actions, work with examples, and prove basic results.
3. Understand rings, integral domains, and fields and results in the theory of rings.
4. Understand the various types of rings, such as polynomial rings, the Gaussian integers, and certain factor rings.

Contents:

Unit-I Definition of a group, examples including matrices, permutation groups, groups of symmetry, roots of unity. Properties of a group, finite and infinite groups.

Unit-II Subgroups and cosets, order of an element, Lagrange theorem, normal subgroups, quotient groups. Detailed look at the group S_n of permutations, cycles and transpositions, even and odd permutations, the alternating group, simplicity of A_n for n.

Unit-III Homomorphisms, kernel, image, isomorphism, the fundamental theorem of group Homomorphisms. Abelian group, cyclic groups, subgroups and quotients of cyclic groups, finite and infinite cyclic groups.

Unit-IV Cayleys theorem on representing a group as a permutation group. Conjugacy classes, centre, class equation, centre of a p-group. Sylow theorems.

Unit-V Definition of a ring, examples including congruence classes modulo n, ideals and Homomorphisms, quotient rings, polynomial ring in one variable over a ring, units, fields, nonzero divisors, integral domains. Rings of fractions, field of fractions of an integral domain.

References:

[1] Serge Lang. *Algebra.* Springer Science & Business Media, 2012.

[2] Nathan Jacobson. *Basic Algebra II.* Freeman, New York, 1989.

[3] David S Dummit and Richard M Foote. *Abstract Algebra.* John Wiley and Sons, Inc, 2004.

[4] Michael Artin. *Algebra.* Pearson College Division, 1991.

3.4 M304: Elementary Number Theory

Learning Outcomes: At the end of the course, the students will be able to :

1. Use continued fractions to develop arbitrarily accurate rational approximations to rational and irrational numbers.
2. Analyze Diophantine equations, i.e. polynomial equations with integer solutions.
3. Know what it means to say that an integer is a quadratic reside modulo an odd prime, and calculate whether this relation is true for a given integer and prime.
4. Know some of the famous classical theorems and conjectures in number theory, such as Fermat's Last Theorem and Goldbach's Conjecture, and describe some of the tools used to investigate such problems.

Contents:

Unit-I Fundamental theorem of arithmetic, divisibility in integers.Prime numbers and infinitude of primes. Infinitude of primes of special types. Special primes like Fermat primes, Mersenne primes, Lucas primes etc. Euclidean algorithm, greatest common divisor, least common multiple.

Unit-II Equivalence relations and the notion of congruences. Wilson's theorem and Fermat's little theorem. Chinese remainder theorem. Continued fractions and their applications. Primitive roots, Euler's Phi function. Sum of divisors and number of divisors, Mobius inversion.

Unit-III Quadratic residues and non-residues with examples. Euler's Criterion, Gauss' Lemma. Quadratic reciprocity and applications. Applications of quadratic reciprocity to calculation of symbols.

Unit-IV Legendre symbol: Definition and basic properties. Fermat's two square theorem, Lagrange's four square theorem.

Unit-V Pythagorean triples. Diophantine equations and Bachet's equation. The duplication formula.

References:

[1] David Burton. *Elementary Number Theory*. McGraw Hill, 2010.

[2] Kenneth H Rosen. *Elementary number theory and its applications*, volume 1. Pearson/Addison Wesley, 2005.

[3] Ivan Niven, Herbert S Zuckerman, and Hugh L Montgomery. *An introduction to the theory of numbers*. John Wiley & Sons, 1991.

3.5 M305 : Computational Mathematics-I

Learning Outcomes: At the end of the course, the students will be able to :

1. Know core programming techniques of Mathematica.
2. Understand fundamentals of Mathematica and its computational efficiency.
3. Use of Mathematica to understand Calculus, Linear Algebra.
4. Use of Mathematica to solve linear and nonlinear equations.

Contents:

Unit-I: Core language and structure: introduction to prgramming, Notation and conversion, Mathematica basic concept, constants, strings, lists, Mathematical expressions.

Unit-II Functional programming: Built- in Functions, user- defined functions, Operation on functions, Recursive functions, Iterative functions, Loops and Flow- control.

Unit-III Two-Dimensional Graphics: Plotting functions of single variable, Three Dimensional Graphics – Plotting functions of two variables, other graphics command, Algebra and trigonometry.

Unit-IV Basic Linear Algebra and Calculus using Mathematica.

Unit-V Numerical Solutions of Linear and Non-linear equations using Mathematica. Developing Programs for each of these methods.

References:

[1] Eugene Don. *Schaum's outline of Mathematica*. McGraw-Hill Professional, 2000.

[2] Kenneth M Shiskowski and Karl Frinkle. *Principles of Linear Algebra with Mathematica*. John Wiley & Sons, 2013.

[3] Selwyn L Hollis. *CalcLabs with Mathematica: Multivariable Calculus*. Thomson Learning, 1998.

[4] Selwyn L Hollis. *Multivariable Calculus*. Brooks/Cole Publishing Company, 2002.

3.6 GL301 : Computational Mathematics-I

Learning Outcomes: At the end of the course, the students will be able to :

1. Know core programming techniques of Mathematica and able to make program for simple problems.
2. Understand fundamentals of Mathematica and its computational efficiency.
3. Use of Mathematica to understand Calculus, Linear Algebra
4. Use of Mathematica to solve linear and nonlinear equations.

Contents: Practicals of computational mathematics laboratory using Mathematica based on syllabus of M305.

References:

[1] Eugene Don. *Schaum's outline of Mathematica.* McGraw-Hill Professional, 2000.

[2] Kenneth M Shiskowski and Karl Frinkle. *Principles of Linear Algebra with Mathematica.* John Wiley & Sons, 2013.

[3] Selwyn L Hollis. *CalcLabs with Mathematica: Multivariable Calculus.* Thomson Learning, 1998.

[4] Selwyn L Hollis. *Multivariable Calculus.* Brooks/Cole Publishing Company, 2002.

3.7 CB301: Essential Mathematics for Chemistry & Biology

Unit-I First Order Differential Equations: Linear Equations, Nonlinear Equations, Separable Equations, Exact Equations, Integrating Factors.

Unit-II Second Order Linear Differential Equations: Fundamental Solutions for the Homogeneous Equation, Linear Independence, Reduction of Order, Homogeneous Equations with Constant Coefficients.

Unit-III Laplace transforms, inverse Laplace transforms, convolution theorem, applications of Laplace transform to solve system of differential equations.

Unit-IV Vector Spaces (finite dimensional, over $\mathbb{R}$ or $\mathbb{C}$. Illustrate concepts with 2- or 3- dimensional examples), Linear Independence, Basis, Dimension, Rank of a Matrix.

Unit-V The matrix Eigen value problems, Secular determinants, Characteristics polynomials, Eigen values and Eigen vectors. Eigen values of real symmetric matrices; Eigen values and Eigen vectors, important properties and examples.

References:

[1] MD Raisinghania. *Ordinary and partial differential equations.* S. Chand Publishing, 2013.

[2] George F Simmons. *Differential equations with applications and historical notes.* CRC Press, 2016.

[3] Marc Lipson and Seymour Lipschutz. *Schaum's Outline of Linear Algebra.* McGraw-Hill Companies, Incorporated, 2018.

[4] S Kumaresan. *Linear algebra: a geometric approach.* PHI Learning Pvt. Ltd., 2000.

4 Semester-IV

4.1 M401:Analysis II

Learning Outcomes: At the end of the course, the students will be able to :

1. Understand the concept and properties of Reimann integration.
2. Able to evaluate improper integrals.
3. Understand function of multi-variables to be continuous and calculate partial derivatives, directional derivatives.
4. Understand the chain rule for functions of two and three variables.
5. Understand the properties of double integrals and calculate multiple integrals.
6. Understand the concept of integration on curves and surfaces.

Contents:

Unit-I Riemann Integration: Definition via upper and lower Riemann sums, basic properties. Riemann integrability, continuous implies f is Riemann integrable, examples of Riemann integrable functions which are not continuous on $[a, b]$. Properties of Riemann Integration.

Unit-II: Improper integrals, power series and elementary functions: Cauchy's condition for existence of improper integrals, test for convergence. Examples: $\int \frac{\sin x}{x} dx$, $\int \cos x^2 dx$, $\int \sin x^2 dx$. Power series and basic properties, continuity of the sum, validity of term by term differentiation. Binomial theorem for arbitrary real coefficients. Elementary transcendental functions e^x, $\sin x$, $\cos x$ and their inverse functions, $\log x$, $\tan^{-1} x$, Gudermannian and other examples.

Unit-III: Linear maps from R^n to R^m, Directional derivative, partial derivative, total derivative, Jacobian, Mean value theorem and Taylor's theorem for several variables, Chain Rule. Parametrized surfaces, coordinate transformations, Inverse function theorem, Implicit function theorem, Rank theorem.

Unit-IV: Critical points, maxima and minima, saddle points, Lagrange multiplier method.

Unit-V: Multiple integrals, Riemann and Darboux integrals, Iterated integrals, Improper integrals, Change of variables. Integration on curves and surfaces, Greens theorem, Differential forms, Divergence, Stokes theorem.

References:

[1] James J Callahan. *Advanced calculus: a geometric view*, volume 1. Springer, 2010.

[2] Terence Tao. *Analysis*. Springer, 2009.

[3] Peter D Lax and Maria Shea Terrell. *Multivariable Calculus with Applications*. Springer, 2017.

[4] Miklós Laczkovich and Vera T Sós. *Real Analysis: Series, Functions of Several Variables, and Applications*, volume 3. Springer, 2017.

[5] Stanley J Miklavcic. *An illustrative guide to multivariable and vector calculus*. Springer Nature, 2020.

[6] Walter Rudin. *Principles of mathematical analysis*. McGraw-hill New York, 1976.

[7] George Pedrick. *A first course in analysis*. Springer Science & Business Media, 1994.

4.2 M402: Algebra II (Linear Algebra)

Learning Outcomes: At the end of the course, the students will be able to :

1. Understand the concept of vector space, subspace and basis.
2. Understand whether or not a given subset of a vector space is a subspace; whether or not a given vector is in the subspace spanned by a set of vectors; and whether given vectors are linearly independent and/or form a basis for a vector space or subspace.
3. Understand whether a mapping between vector spaces is linear, and if so calculate the matrix of the mapping with respect to given bases.
4. Understand whether the vectors are orthogonal and/or orthonormal and Gram-Schmidt process.
5. Understand Quadratic form, Jordan and rational canonical forms.

Contents:

Unit-I Vector spaces over a field, subspaces, quotient spaces. Span and linear independence, basis, dimension.

Unit-II: Linear maps and their correspondence with matrices with respect to given bases, change of bases.

Unit-III: Eigen-values, eigen-vectors, eigen-spaces, characteristic polynomial, Cayley-Hamilton theorem.

Unit-IV: Bilinear forms, inner product spaces, Gram-Schmidt process, diagonalization, spectral theorem.

Unit-V: Quadratic form, Jordan and rational canonical forms. System of linear equations.

References:

[1] Ramji Lal. *Algebra 2: Linear Algebra, Galois Theory, Representation Theory, Group Extensions and Schur Multiplier*. Springer, 2017.

[2] S Kumaresan. *Linear algebra: a geometric approach*. PHI Learning Pvt. Ltd., 2000.

[3] Marc Lipson and Seymour Lipschutz. *Schaum's Outline of Linear Algebra*. McGraw-Hill Companies, Incorporated, 2018.

[4] Serge Lang. *Linear algebra*. Springer Berlin, 1987.

[5] Kenneth Hoffman and Ray Kunze. *Linear Algebra, Prentice-Hall*. Inc., Englewood Cliffs, New Jersey, 1971.

4.3 M403 : Introduction to Differential Equations

Learning Outcomes: At the end of the course, the students will be able to :

1. Understand concept of Laplace and inverse Laplace transformations and its applications.
2. Understand the concept of partial differential equations of first order and various methods of solutions.
3. Able to solve linear partial differential equations of second and higher order.
4. Understand the concept of calculus of variations and variational problems.
5. Able to solve variational problems with fixed and moving boundaries.

Contents:

Unit-I Laplace Transformation- Linearity of the Laplace transformation. Existence theorem for Laplace transforms. Laplace transforms of derivatives and integrals. Shifting theorems. Differentiation and integration of transforms. Convolution theorem. Solution of integral equations and systems of differential equations using the Laplace transformation.

Unit-II Partial differential equations of the first order. Lagrange's solution, Some special types of equations which can he solved easily by methods other than the general method. Charpit's general method of solution.

Unit-III Partial differential equations of second and higher orders, Classification of linear partial differential equations of second order, Homogencous and non-homogeneous equations with constant coefficients, Partial differential equations reducible to equations with constant coefficients, Monge's methods.

Unit-IV Calculus of Variations- Variational problems with fixed boundaries- Euler's equation for functionals containing first order derivative and one independent variable, Extemals, Functionals dependent on higher order derivatives. Functionals dependent on more than one independent variable. Variational problems in parametric form, invariance of Euler's equation under coordinates transformation.

Unit-V Variational Problems with Moving Boundaries- Functionals dependent on one and two functions. One sided variations. Sufficient conditions for an Extremum- Jacobi and Legendre conditions, Second Variation. Variational principle of least action.

References:

[1] AK Nandakumaran, PS Datti, and Raju K George. *Ordinary differential equations: Principles and applications.* Cambridge University Press, 2017.

[2] S.L. Ross. *Introduction to Ordinary Differential Equations.* Wiley, 1989.

[3] George F Simmons. *Differential equations with applications and historical notes.* CRC Press, 2016.

[4] MD Raisinghania. *Ordinary and partial differential equations.* S. Chand Publishing, 2013.

[5] AS Gupta. *Calculus of variations with applications.* PHI Learning Pvt. Ltd., 1996.

[6] Erwin Kreyszig. *Advanced Engineering Mathematics 9th Edition with Wiley Plus Set.* John Wiley & Sons, 2007.

4.4 M404: Topology I

Learning Outcomes: At the end of the course, the students will be able to :

1. Understand the generalization of concept of distance in the form of metric.
2. Understand basic properties of metric space and inequalities on metric space.
3. Understand topology generated by metric space, concept of open/closed ball/set and their properties.
4. Comprehend Hausdorff property of a metric space, equivalence of metrics, continuity and uniform continuity.
5. Understand the concept and properties of compactness and properties of countable compactness in metric spaces.

Contents:

Unit-I Metric spaces: Definition and basic examples. The discrete metric on any set. $\mathbb{R}$ and $\mathbb{R}^n$ with Euclidean metrics, Cauchy-Schwarz inequality, definition of a norm on a finite dimensional $\mathbb{R}$-vector space and the metric defined by a norm. The set $C[0,1]$ with the metric given by $\sup|f(t) - g(t)|$, metric subspaces, examples.

Unit-II Topology generated by a metric: Open and closed balls, open and closed sets, complement of an open (closed) set, arbitrary unions (intersections) of open (closed) sets, finite intersections (unions) of open (closed) sets, open (closed) ball is an open (closed) set, properties of open sets

Unit-III Hausdorff property of a metric space. Equivalence of metrics, examples, the metrics on R^2 given by $|x_1 - y_1| + |x_2 - y_2|$ (resp. $max|x_1 - y_1|, |x_2 - y_2|$ is equivalent to the Euclidean metric, the shapes of open balls under these metrics. Limit points, isolated points, interior points, closure, interior and boundary of a set, dense and nowhere dense sets.

Unit-IV Continuous maps: epsilon-delta definition and characterization in terms of inverse images of open (resp. closed) sets, composite of continuous maps, point-wise sums and products of continuous maps into $\mathbb{R}$; homomorphism, isometry, an isometry is a homomorphism but not conversely, uniformly continuous maps. examples.

Complete metric spaces: Cauchy sequences and convergent sequences, a subspace of a complete metric space is complete if and only if it is closed, Cantor intersection theorem, Baire category theorem and its applications, completion of a metric space.

Unit-V Compactness for metric spaces: Bolzano-Weierstrass property, the Lebesgue number for an open covering, sequentially compact and totally bounded metric spaces, Heine-Borel theorem, compact subsets of $\mathbb{R}$; a continuous map from a compact metric space is uniformly continuous.
Connectedness: Definition, continuous image of a connected set is connected, characterization in terms of continuous maps into the discrete space $\mathbb{N}$, connected subsets of $\mathbb{R}$; intermediate value theorem as a corollary, countable (arbitrary) union of connected sets, connected components.

References:

[1] Edward Thomas Copson. *Metric spaces.* Cambridge University Press, 1988.

[2] Robert Herman Kasriel. *Undergraduate topology.* WB Saunders Company, 1971.

[3] W.R. Wade. *Introduction to Analysis: Pearson New International Edition.* Pearson Education, Limited, 2013.

[4] George F Simmons. *Introduction to topology and modern analysis.* Tata McGraw-Hill, 1963.

[5] Wilson A Sutherland. *Introduction to metric and topological spaces.* Oxford University Press, 2009.

4.5 G401: Statistical Techniques and applications

Learning Outcomes: At the end of the course, the students will be able to :

1. Understand how general nature of statistics, various graphical representation of data.
2. Understand basic probability theory and theoretical distributions.
3. Apply the theoretical distributions to real life problems.
4. Understand the relationships between distributions related to the normal distribution.
5. Understand chi-squared goodness of fit tests for samples from specified population models.
6. Understand the concepts of populations and samples and different kinds of variables, concepts of hypothesis testing and concepts of estimation.

Contents:

Unit-I General nature and scope of statistical methods: Collection and classification of data; different types of diagrams to represent statistical data; frequency distribution and related graphs and charts. Central tendency: Its measure and their uses. Dispersion: Its measure and their uses, Moments; skewness and kurtosis. Scatter diagram.

Unit-II: Elementary idea of probability: Events and Probabilities, Assignments of probabilities to events, addition and multiplication theorems; statistical independence and conditional probability; repeated trials; mathematical expectation; Random events and variables, Probability Axioms and Theorems. Probability distributions and properties: Discrete, Continuous and Empirical distributions.

Unit-III: Expected values: Mean, Variance, Skewness, Kurtosis, Moments and Characteristics Functions. Types of probability distributions: Binomial, Poisson, Normal, Gamma, Exponential, Chi-squared, Log-Normal, Student's t, F distributions, Central Limit Theorem.

Unit-IV: Monte Carlo techniques: Methods of generating statistical distributions: Pseudorandom numbers from computers and from probability distributions, Applications. Parameter inference: Given prior discrete hypotheses and continuous parameters. Maximum likelihood method for parameter inference.

Unit-V: Hypothesis tests: Single and composite hypothesis, Goodness of fit tests, P-values, Chi-squared test, Likelihood Ratio, Kolmogorov- Smirnov test, Confidence Interval. Covariance and Correlation, Analysis of Variance and Covariance. Illustration of statistical techniques through hands-on use of computer program R.

References:

[1] SC Gupta and VK Kapoor. *Fundamentals of mathematical statistics.* Sultan Chand & Sons, 2020.

[2] Stephen Kokoska. *Introductory Statistics: A Problem-Solving Approach.* Macmillan, 2008.

[3] Geoffrey Grimmett and David Stirzaker. *Probability and random processes.* Oxford university press, 2020.

[4] Sheldon M Ross. *Introduction to probability and statistics for engineers and scientists.* Academic press, 2020.

[5] Michael Akritas. *Probability and Statistics with R.* New York: Pearson, 2015.

[6] Dieter Rasch, Rob Verdooren, and Jürgen Pilz. *Applied Statistics: Theory and Problem Solutions with R.* John Wiley & Sons, 2019.

4.6 GL401: Computational Laboratory and Numerical Methods (Using Python)

Learning Outcomes: At the end of the course, the students will be able to :

1. Understand fundamentals of Python programming language.
2. Understand programming techniques using Python.
3. Able to implement basic numerical analysis using Python.
4. Able to implement matrix algebra in Python.

Contents:

Unit-I Introduction to Python : Datatypes – Int, Float, Boolean, Sting and list, Variable expressions, Statements, Precedence of operators comments, module functions and its uses, flow of exection parameters and arguments.

Unit-II Control flow, loops : conditionals, Boolean values and operators, conditional (if) alternative (if-else), Chained conditional (if-elif-else), Iteration while, for, Break, continue, functions, Arrays, List, Tuples, Dictionaries.

Unit-III Machine representation and precision, Error and it's sources , propagation and Analysis; Error in summation, Stability in numerical analysis, Linear algebraic equations, Gaussian elimination, direct Triangular Decomposition, matrix inversion. One should understand how to analyze whether a calculation is limited by the algorithm or round-off error. Single/double precision.

Unit-IV Basic tools for numerical analysis in science : Solution of algebraic functions–Fixed point method, Newton-Raphson method, Secant method. Numerical Integration – Rectangular method, trapezoidal method.Lagrang interpolation.

Unit-V Matrix Algebra: Approximate solution of a set of linear simultaneous equations by Gauss- Side iteration method. Exact solution by Gaussian elimination. Inversion of a matrix by Gaussian elimination. Determining all the eigen values of a real symmetric matrix by Householder's method of tri-diagonalization followed by Q R factorization of the tridiagonalized matrix.

References:

[1] Chris Roffey. *Cambridge IGCSE® and O Level Computer Science Programming Book for Python.* Cambridge University Press, 2017.

[2] Bhajan Singh Grewal and Grewal. JS. *Numerical methods in Engineering and Science.* Khanna Publishers, 1996.

4.7 GL402: Statistical Techniques Laboratory

Learning Outcomes: At the end of the course, the students will be able to :

1. Understand basic programming techniques of R/R-Studio.
2. Plot one and two-dimensional data in an appropriate way and interpret such plots.
3. Calculate summary statistics for a set of data.
4. Carry out tests and calculate confidence intervals for normal samples with known variance.
5. Carry out other hypothesis tests.

Contents: Practical of applied statistics using R programming language based on syllabus of G-401.

References:

[1] Chris Roffey. *Cambridge IGCSE® and O Level Computer Science Programming Book for Python.* Cambridge University Press, 2017.

[2] Bhajan Singh Grewal and Grewal. JS. *Numerical methods in Engineering and Science.* Khanna Publishers, 1996.

5 Semester-V

5.1 M501 : Analysis III (Measure Theory and Integration)

Learning Outcomes: At the end of the course, the students will be able to :

1. Understand the concept of measure space, Lebesgue outer measure on real line.
2. Understand the concept of measurable function, types of convergence and integrable functions.
3. Understand convergence theorems and its consequences.
4. Comprehend the product measure, Fubini's theorem and inequalities on L^p and L^∞ spaces.

Contents:

Unit-I Sigma algebra of sets, measure spaces. Lebesgue outer measure on the real line. Measurable set in the sense of Caratheodory. Translation invariance of Lebesgue measure. Existence of a non-Lebesgue measurable set. Cantor set- uncountable set with measure zero.

Unit-II: Measurable functions, types of convergence of measurable functions. The Lebesgue integral for simple functions, nonnegative measurable functions and Lebesgue integrable function, in general

Unit-III Convergence theorems- monotone and dominated convergence theorems.

Unit-IV Comparison of Riemann and Lebesgue integrals. Riemanns theorem on functions which are continuous almost everywhere.

Unit-V The product measure and Fubini's theorem. The L^p spaces and the norm topology. Inequalities of Holder and Minkowski. Completeness of L^p and L^∞ spaces.

References:

[1] P.K. Jain, P.K. Jain, and V.P. Gupta. *Lebesgue Measure and Integration.* A Halsted press book. Wiley, 1986.

[2] S. Shirali. *A Concise Introduction to Measure Theory.* Springer International Publishing, 2019.

[3] C.D. Aliprantis and O. Burkinshaw. *Principles of Real Analysis.* Academic Press, 2008.

[4] I.K. Rana. *An Introduction to Measure and Integration.* Alpha Science international, 2005.

[5] J. Yeh. *Problems and Proofs in Real Analysis: Theory of Measure and Integration.* World Scientific, 2014.

[6] S.G. Krantz. *Elementary Introduction to the Lebesgue Integral.* CRC Press, 2018.

[7] HL Royden and PM Fitzpatrick. *Real analysis 4th Edition.* Printice-Hall Inc, Boston, 2010.

5.2 M502 : Algebra III (Galois Theory)

Learning Outcomes: At the end of the course, the students will be able to :

1. Understand the concept of prime and maximal ideals.
2. Understand field extensions and associated properties.
3. Understand finite Galois extension.
4. Understand solvability by radicals and extension of finite fields.

Contents:

Unit-I Prime and maximal ideals in a commutative ring and their elementary properties.

Unit-II: Field extensions, prime fields, characteristic of a field, algebraic field extensions, finite field extensions, splitting fields, algebraic closure, separable extensions, normal extensions.

Unit-III: Finite Galois extensions, Fundamental Theorem of Galois Theory.

Unit-IV: Solvability by radicals.

Unit-V: Extensions of finite fields.

References:

[1] R. Lal. *Algebra 1: Groups, Rings, Fields and Arithmetic.* Infosys Science Foundation Series. Springer Singapore, 2017.

[2] R. Lal. *Algebra 2: Linear Algebra, Galois Theory, Representation theory, Group extensions and Schur Multiplier.* Infosys Science Foundation Series. Springer Nature Singapore, 2017.

[3] J.A. Gallian. *Contemporary Abstract Algebra.* Textbooks in mathematics. CRC, Taylor & Francis Group, 2020.

[4] P.B. Bhattacharya, S.K. Jain, and S.R. Nagpaul. *Basic Abstract Algebra.* Basic Abstract Algebra. Cambridge University Press, 1994.

[5] David Steven Dummit and Richard M Foote. *Abstract algebra.* Wiley Hoboken, 2004.

[6] Nathan Jacobson. *Basic algebra 1.* Courier Corporation, 2012.

[7] Nathan Jacobson. *Lectures in Abstract Algebra: II. Linear Algebra.* Springer Science & Business Media, 2013.

[8] Serge Lang. *Algebra.* Springer Science & Business Media, 2012.

5.3 M503: Topology II

Learning Outcomes: At the end of the course, the students will be able to :

1. Understand the Topological spaces, order topology and product topology.
2. Understand compactness for general topological spaces.
3. Understand Separation axioms, countability axioms Urysohn's metrization theorem.
4. Understand weak topology, coherent topology and embeddings.
5. Understand completely regular spaces and compactification.

Contents:

Unit -I General topological spaces, stronger and weaker topologies, continuous maps, homomorphisms, bases and subbases, finite products of topological spaces.

Unit-II Compactness for general topological spaces: Finite sub-coverings of open coverings and finite intersection property, continuous image of a compact set is compact, compactness and Hausdorff property.

Unit-III: Basic Separation axioms and first and second countability axioms. Examples. Products and quotients. Tychonoff's theorem. Product of connected spaces is connected.

Unit-IV Weak topology on X induced by a family of maps $f_\alpha : X \to X_\alpha$ where each $X\alpha$ is a topological space. The coherent topology on Y induced by a family of maps $g_\alpha : Y_\alpha \to Y$ are given topological spaces. Examples of quotients to illustrate the universal property such as embeddings of RP^2 and the Klein's bottle in R^4.

Unit-V Completely regular spaces and its embeddings in a product of intervals. Compactification, Alexandroff and Stone-Cech compactifications. Normal spaces and the theorems of Urysohn and Tietze. The metrization theorem of Urysohn.

References:

[1] Kapil D Joshi. *Introduction to general topology.* New Age International, 1983.

[2] F Simmons George. Topology and modern analysis.(1963).

[3] James Munkres. Topology james munkres, second edition. 1999.

[4] John B Conway. *A course in point set Topology.* Springer, 2014.

5.4 M504:Probability Theory

Learning Outcomes: At the end of the course, the students will be able to :

1. Understand the sample space for simple experiments and calculate probabilities in straightforward instances of these types of experiment.
2. Understand the Kolmogorov axioms for probability.
3. Define and recognise independent events. Use independence to calculate probabilities.
4. Understand conditional probability, random variable and the probability mass function of a discrete random variable.
5. Understand the main properties of Bernoulli, binomial, geometric, and Poisson random variables.
6. Understand Central limit theorem, characteristic functions, moment generating functions.
7. Understand Random walks, Markov Chains and their properties.
8. Understand conditional expectations, its properties.

Contents:

Unit-I Probability as a measure, Probability space, conditional probability, independence of events, Bayes formula. Random variables, distribution functions, expected value and variance. Standard Probability distributions: Binomial, Poisson and Normal distribution.

Unit-II Borel-Cantelli lemmas, zero-one laws. Sequences of random variables, convergence theorems, Various modes of convergence. Weak law and the strong law of large numbers.

Unit-III Central limit theorem: DeMoivre-Laplace theorem, weak convergence, characteristic functions, inversion formula, moment generating function.

Unit-IV Random walks, Markov Chains, Recurrence and Transience.

Unit-V Conditional Expectation, Martingales.

References:

[1] Geoffrey Grimmett and David Stirzaker. *Probability and random processes.* Oxford university press, 2020.

[2] Marek Capinski and Tomasz Jerzy Zastawniak. *Probability through problems.* Springer Science & Business Media, 2013.

[3] Joseph K Blitzstein and Jessica Hwang. *Introduction to probability.* Crc Press Boca Raton, FL, 2015.

[4] Jeffrey S Rosenthal. *First Look At Rigorous Probability Theory, A.* World Scientific Publishing Company, 2006.

[5] Kai Lai Chung and Farid AitSahlia. *Elementary probability theory: with stochastic processes and an introduction to mathematical finance.* Springer Science & Business Media, 2006.

5.5 PM501: Numerical Analysis

Learning Outcomes: At the end of the course, the students will be able to :

1. Understand types of errors, its sources and propagation analysis.
2. Understanding various techniques to solve algebraic and transcendental equations (bisection, Newton's, secant etc.).
3. Understand the concept of interpolation and extrapolation using various techniques (Lagrange's, Newton's farward/backward/divided difference methods).
4. Understand various techniques of numerical integration (Trapezoidal, Simpson's, Gaussian quadrature)
5. Understand various techniques of numerical differentiation (Euler's, Runge -Kutta, predictor - corrector).
6. Understand least square problems (linear and nonlinear).

Contents:

Unit-I Error, its sources, propagation and analysis; Errors in summation, stability in numerical analysis. Linear algebraic equations: Gaussian elimination, direct triangular decomposition, matrix inversion.

Unit-II Root finding: review of bisection method, Newton's method and secant method; real roots of polynomials, Laguerre's method. Matrix eigenvalue problems: Power method, eigenvalues of real symmetric matrices using Jacobi method, applications.

Unit-III Interpolation theory: Polynomial interpolation, Newton's divided differences, forward differences, interpolation errors, cubic splines. Approximation of functions: Taylor's theorem, remainder term; Least squares approximation problem, Orthogonal polynomials.

Unit-IV Numerical integration: review of trapezoidal and Simpson's rules, Gaussian quadrature; Error estimation. Numerical differentiation. Monte Carlo methods.

Unit-V Least squares problems: Linear least squares, examples; Non - linear least squares. Ordinary differential equations: stability, predictor - corrector method, Runge - Kutta methods, boundary value problems, basis expansion methods, applications. Eigenvalue problems for differential equations, applications.

References:

[1] Bhajan Singh Grewal and Grewal. JS. *Numerical methods in Engineering and Science*. Khanna Publishers, 1996.

[2] Kendall E Atkinson. *An introduction to numerical analysis*. John wiley & sons, 2008.

[3] Amos Gilat. *MATLAB: an introduction with applications*. John Wiley & Sons, 2004.

[4] Richard Hamming. *Numerical methods for scientists and engineers*. Courier Corporation, 2012.

[5] Y Vetterlillg, V Press, S Teukolsky, and B Flannery. *Numerical Recipes in C*. Cambridge University Press, 2000.

5.6 PML501: Numerical Methods Laboratory

Learning Outcomes: At the end of the course, the students will be able to :

1. Understand fundamentals of programming language MATLAB.
2. Able to make programs for simple problems using m-file.
3. Able to implement estimation of errors, its propagation analysis.
4. Able to solve algebraic and transcendental equations.
5. Implement various techniques of numerical solution of differential equations.
6. Implement least square problems (linear and nonlinear).

Contents:

Unit-I: Starting Matlab, Matlab windows, Working in the command window, arithmetic Operations with scalars, Math Built-in function, Designing Scalar variables, Useful commands for managing variables, Script files.

Unit-II Creating Arrays, Mathematical Operations with Arrays, Two-Dimensional Plots, Relational and logical Operators, Conditional statement, loops , Nested Loops and Nested Conditional Statements, The break and continue command.

Unit-III Creating a function file, Structure of a function file, Local and Global variable, Polynomials - value of a polynomial, Roots of a polynomial, Addition , multiplication and Division of polynomials, Derivative of polynomials, Curve Fitting with polynomials, Interpolation.

Unit-IV Solution of Algebraic and Transcendental Equation, Basic properties of equations, Synthetic Division of a polynomial by a linear expression, Graphics Solution of equations, Bisection Method , Secant Method, Newton Raphson Method, Mullers Method, Gauss elimination Method, Gauss Jordan Method.

Unit-V Numerical soluiton of ordinary Differential equations: Introduction, Picard Method, Euler's Method, Taylor's Series Method, Runge kutta Method, Boundary value problems, Eigen value problems

References:

[1] Bhajan Singh Grewal and Grewal. JS. *Numerical methods in Engineering and Science*. Khanna Publishers, 1996.

[2] Amos Gilat. *MATLAB: an introduction with applications*. John Wiley & Sons, 2004.

[3] Richard Hamming. *Numerical methods for scientists and engineers*. Courier Corporation, 2012.

6 Semester - VI

6.1 M601: Analysis IV (Complex Analysis)

Learning Outcomes: At the end of the course, the students will be able to :

1. Find all complex solutions of a simple polynomial, indicating their position in the Argand diagram.
2. Derive the Cauchy-Riemann equations for a given function and understand the concept of entire, holomorphic and harmonic functions.
3. Use the theory of Möbius transformations to solve simple problems.
4. Understand the Taylor and Laurent series of a functions about a given point, contour integral of a complex function.
5. State the Residue Theorem and apply it when appropriate to calculate a contour integral.
6. Understand the concept of Morera's theorem, Liouville's theorem and Fundamental theorem of algebra.

Contents:

Unit-I Complex numbers and Riemann sphere. Mobius transformations.

Unit-II Analytic functions. Cauchy-Riemann conditions, harmonic functions, Elementary functions, Power series, Conformal mappings.

Unit-III Contour integrals, Cauchy theorem for simply and multiply connected domains. Cauchy integral formula, Winding number

Unit-IV Morera's theorem. Liouville's theorem, Fundamental theorem of Algebra. Zeros of an analytic function and Taylors theorem. Isolated singularities and residues, Laurent series, Evaluation of real integrals.

Unit-V Zeros and Poles, Argument principle. Rouchs theorem.

References:

[1] Saminathan Ponnusamy and Herb Silverman. *Complex variables with applications.* Springer, 2006.

[2] D Martin and LV Ahlfors. *Complex analysis.* New York: McGraw-Hill, 1966.

[3] Bruce P Palka. *An introduction to complex function theory.* Springer Science & Business Media, 1991.

[4] Ruel Churchill and James Brown. *Complex Variables and Applications.* McGraw Hill, 2014.

[5] Endre Pap. *Complex analysis through examples and exercises.* Springer Science & Business Media, 1999.

[6] Dennis Spellman. *Schaum's Outline of Complex Variables.* McGraw-Hill, New York, 2009.

6.2 M602: Algebra IV (Rings and Modules: Some Structure Theory)

Learning Outcomes: At the end of the course, the students will be able to :

1. Be familiar with rings and fields, and understand the structure theory of modules over a Euclidean domain along with its implications.
2. Understand the concept of External and internal direct sums of modules and Tensor product of modules over a commutative ring.
3. Understand the concept of elementary properties of projective and injective modules over a commutative ring.
4. Understand the concept of Structure of finitely generated modules over a PID, aimple modules over a not necessarily commutative ring.

5. Understand the concept of Jordan- Holder Theorem, Schur's lemma. Semi simple modules.

Contents:

Unit-I Modules, submodules, quotient modules, homomorphisms.

Unit-II External and internal direct sums of modules. Tensor product of modules over a commutative ring. Functor properties of Homomorphism.

Unit-III Definitions and elementary properties of projective and injective modules over a commutative ring.

Unit-IV Structure of finitely generated modules over a PID. Applications to matrices and linear maps over field:

Unit-V Simple modules over a not necessarily commutative ring, modules of finite length, Jordan- Holder Theorem, Schur's lemma. Semisimple modules.

References:

[1] Phani Bhushan Bhattacharya, Surender Kumar Jain, and SR Nagpaul. *Basic abstract algebra.* Cambridge University Press, 1994.

[2] David Steven Dummit and Richard M Foote. *Abstract algebra*, volume 3. Wiley Hoboken, 2004.

[3] Nathan Jacobson. *Basic algebra I.* Courier Corporation, 2012.

[4] Nathan Jacobson. *Lectures in Abstract Algebra: II. Linear Algebra.* Springer Science & Business Media, 2013.

[5] Serge Lang. *Algebra.* Springer Science & Business Media, 2012.

6.3 M603 : Partial Differential Equations

Learning Outcomes: At the end of the course, the students will be able to :

1. Use partial differential equations to model certain physical systems.
2. Choose the most appropriate method to solve a range of partial differential equations.
3. Understand the limitations of analytical solution methods and understand where numerical methods are required.
4. Understand the wave equation and the Cauchy problem for the wave equation.
5. Understand the Fourier methods for solving initial boundary value problems.
6. Understand the Detailed analysis of the Laplace and Poisson's equations, Green's function for the Laplacian and its basic properties.

Contents:

Unit-I Generalities on the origins of partial differential equations. Generalities on the Cauchy problem for a scalar linear equation of arbitrary order. The concept of characteristics. The Cauchy-Kowalevsky theorem and the Holmgren's uniqueness theorem. First order partial differential equations and their solutions.

Unit-II Quasilinear first order scalar partial differential equations and the method of characteristics. Detailed discussion of the inviscid Burger's equation illustrating the formation of discontinuities in finite time. The fully nonlinear scalar equation and Eikonal equation. The Hamilton-Jacobi equation.

Unit-III Detailed analysis of the Laplace and Poisson's equations. Green's function for the Laplacian and its basic properties. Integral representation of solutions and its consequences such as the analyticity of solutions. The mean value property for harmonic functions and maximum principles. Harnack inequality.

Unit-IV The wave equation and the Cauchy problem for the wave equation. The Euler-Poisson Darboux equation and integral representation for the wave equation in dimensions two and three. Properties of solutions such as finite speed of propagation. Domain of dependence and domain of inuence.

Unit-V The Cauchy problem for the heat equation and the integral representation for the solutions of The Cauchy problem for Cauchy data satisfying suitable growth restrictions. Infinite speed of propagation of signals. Example of non-uniqueness. Fourier methods for solving initial boundary value problems

References:

[1] AK Nandakumaran and PS Datti. *Partial Differential Equations: Classical Theory with a Modern Touch.* Cambridge University Press, 2020.

[2] MD Raisinghania. *Ordinary and partial differential equations.* S. Chand Publishing, 2013.

[3] A Weinstein. *R. Courant and D. Hilbert, Methods of mathematical physics.* American Mathematical Society, 1954.

[4] Robert C McOwen. *Partial differential equations: methods and applications.* Pearson, 2004.

6.4 M604: Ordinary Differential Equations

Learning Outcomes: At the end of the course, the students will be able to :

1. Identify and solve first-order ordinary differential equations.
2. Identify and solve linear second-order non-homogeneous differential equations with constant coefficients and associated IVPs and BVPs.
3. Identify and solve general systems of first-order linear differential equations with constant coefficients using matrix operations.
4. Understand the Series solutions of ordinary differential equations and a detailed analytic study of the differential equations of Bessel and Legendre.
5. Understand the Sturm comparison and separation theorems and regular Sturm-Liouville problems, Wronskians and its basic properties, The Abel Liouville formula.

Contents:

Unit-I Basic existence and uniqueness of systems of ordinary differential equations satisfying the Lipschitz's condition. Examples illustrating non-uniqueness when Lipschitz or other relevant conditions are dropped. Gronwall's lemma and its applications to continuity of the solutions with respect to initial conditions. Smooth dependence on initial conditions and the variational equation. Maximal interval of existence and global solutions. Proof that if (a, b) is the maximal interval of existence and a < 1 then the graph of the solution must exit every compact subset of the domain on the differential equation.

Unit-II Linear systems and fundamental systems of solutions. Wronskians and its basic properties. The Abel Liouville formula. The dimensionality of the space of solutions. Fundamental matrix. The method of variation of parameters.

Unit-III Linear systems with constant coefficients and the structure of the solutions. Matrix exponentials and methods for computing them. Solving the in-homogeneous system.

Unit-IV Second order scalar linear differential equations. The Sturm comparison and separation theorems and regular Sturm-Liouville problems.

Unit-V Series solutions of ordinary differential equations and a detailed analytic study of the differential equations of Bessel and Legendre.

References:

[1] AK Nandakumaran, PS Datti, and Raju K George. *Ordinary differential equations: Principles and applications.* Cambridge University Press, 2017.

[2] S.L. Ross. *Introduction to Ordinary Differential Equations.* Wiley, 1989.

[3] George F Simmons. *Differential equations with applications and historical notes.* CRC Press, 2016.

[4] MD Raisinghania. *Ordinary and partial differential equations.* S. Chand Publishing, 2013.

6.5 M605: Numerical Analysis of Partial Differential Equations

Learning Outcomes: At the end of the course, the students will be able to :

1. Understand various types of PDEs and their classification.
2. Understand and apply Finite difference method to solve PDEs.
3. Understand and apply Finite volumes method to solve PDEs.
4. Understand and apply Spectral method to solve PDEs.
5. Explore convergence analysis of various methods to solve PDEs.

Contents:

Unit-I Partial Differential Equations, Classification, Heat, Wave, Laplace Equations, Elliptical problems.

Unit-II Finite differences for the Two-dimensional Poisson Equation, Convergence Analysis, Room Temperature Simulation using Finite Differences.

Unit-III Finite volumes for a general Two-dimensional Diffusion Equation, Boundary conditions, Relation between Finite Volumes and Finite differences, Finite Volume method are not Consistent, Convergence Analysis.

Unit-IV Spectral Method Based on Fourier series, Spectral Method with Discrete Fourier series, Convergence Analysis, Spectral Method Based on Chebyshev Polynomials.

Unit-V Strong form, Weak or variation form, and Minimization, Discretization, More General Boundary conditions, Convergence Analysis, Generalization to Two - Dimensions.

References:

[1] Martin J Gander and Felix Kwok. *Numerical analysis of partial differential equations using maple and MATLAB.* SIAM, 2018.

[2] Matthew P Coleman. *An introduction to partial differential equations with MATLAB.* CRC Press, 2016.

[3] Jichun Li and Yi-Tung Chen. *Computational partial differential equations using MATLAB®.* Crc Press, 2019.

6.6 ML601: Computational Mathematics Laboratory-III (Numerical Analysis of PDE using Matlab)

Learning Outcomes: At the end of the course, the students will be able to :

1. Use matlab to solve PDE.
2. Understand the Numerical Analysis of PDE using Matlab.
3. Apply finite element method, finite difference method to solve PDEs.
4. Apply the Spectral Method Based on Fourier series, Spectral Method with Discrete Fourier series.
5. Understand the Room Temperature Simulation using Finite Differences.

Contents: Practicals of numerical analysis of partial differential equations using Matlab based on syllabus of M605.

References:

[1] Martin J Gander and Felix Kwok. *Numerical analysis of partial differential equations using maple and MATLAB*. SIAM, 2018.

[2] Matthew P Coleman. *An introduction to partial differential equations with MATLAB*. CRC Press, 2016.

[3] Jichun Li and Yi-Tung Chen. *Computational partial differential equations using MATLAB®*. Crc Press, 2019.

7 Semester - VII

7.1 M701 : Functional Analysis

Learning Outcomes: At the end of the course, the students will be able to :

1. Understand basic idea of a normed linear spaces and operators on normed linear space.
2. Understand Open Mapping theorem, Hahn-Banach Theorem and their applications.
3. To learn to recognize the fundamental properties of normed spaces and of the transformations between them.
4. Be acquainted with the statement of the Hahn-Banach theorem and its corollaries.
5. Understand the notions of dot product and Hilbert space.
6. Understand the statements and proofs of important theorems and be able to explain the key steps in proofs, sometimes with variation.

Contents:

Unit-I Normed linear spaces. Riesz lemma. Heine-Borel theorem. Continuity of linear maps. Hahn-Banach extension and separation theorems

Unit-II Banach spaces. Subspaces, product spaces and quotient spaces. Standard examples of Banach spaces like l^1,L^1,etc.Uniform boundedness principle. Closed graph theorem. Open mapping theorem. Bounded inverse theorem.

Unit-III Spectrum of a bounded operator. Eigenspectrum. Gelfand-Mazur theorem and spectral radius formula. Dual spaces. Transpose of a bounded linear map. Standard examples.

Unit-IV Hilbert spaces. Bessel inequality, Riesz-Schauder theorem, Fourier expansion, Parseval's formula.

Unit-V In the framework of a Hilbert space: Projection theorem. Riesz representation theorem. Uniqueness of Hahn-Banach extension.

References:

[1] S kumaresan and D Sukumar. *Functional Analysis A first course*. Narosa, 2020.

[2] John B Conway. *A course in functional analysis*. Springer, 2019.

[3] Caspar Goffman and George Pedrick. *A first course in functional analysis*. American Mathematical Soc., 2017.

[4] E Kreyszig. Introductory functional analysis with applications, johnwiley & sons inc. *New York-Chichester-Brisbane-Toronto*, 1978.

[5] Balmohan Vishnu Limaye. *Functional analysis*. New Age International, 1996.

[6] Angus Ellis Taylor and David C Lay. *Introduction to functional analysis*, volume 1. Wiley New York, 1958.

7.2 M702: Discrete Mathematics

Learning Outcomes: At the end of the course, the students will be able to :

1. Understand and apply the concept of permutation and combinations.
2. Understand Binomial and multi-nomial theorems.
3. Understand the concept of formal language and grammar.
4. Understand the concept of finite state machine.
5. Understand the concept of analysis of algorithms.
6. Understand and apply the concept of recurrence relations and recursive algorithms.
7. Understand Boolean Algebra.

Contents:

Unit-I: Combinatorics: Permutations and combinations. Linear equations and their relation to distribution into boxes. Distributions with repetitions and non-repetitions. Combinatorial derivation of these formulae. Pigeonhole Principle and applications.

Unit-II Binomial and multinomial theorems. Inclusion-Exclusion Principle and Applications. Computability and Formal Languages - Ordered Sets, Languages. Phrase Structure Grammars. Types of Grammars and Languages.

Unit-III Finite State Machines - Equivalent Machines. Finite State Machines as Language Recognizers. Analysis of Algorithms - Time Complexity. Complexity of Problems. Discrete Numeric Functions and Generating Functions.

Unit-IV Recurrence Relations and Recursive Algorithms - Linear Recurrence Relations with constant coefficients. Homogeneous Solutions. Particular Solution. Total Solution. Solution by the Method of Generating Functions.

Unit-V Boolean Algebras - Lattices and Algebraic Structures. Duality, Distributive and Complemented Lattices. Boolean Lattices and Boolean Algebras, Boolean Functions and Exprers. Calculus. Design and Implementation of Digital Networks. Switching Circuits.

References:

[1] Kenneth H Rosen. *Discrete mathematics and its applications.* McGraw-Hill, 2012.

[2] C Vasudev. *Graph theory with applications.* New Age International, 2006.

[3] Chung Laung Liu. *Elements of discrete mathematics.* McGraw-Hill, Inc., 1985.

[4] Richard Johnsonbaugh. *Discrete mathematics.* Pearson, 2009.

[5] Willem Conradie and Valentin Goranko. *Logic and discrete mathematics: a concise introduction.* John Wiley & Sons, 2015.

[6] Narsingh Deo. *Graph theory with applications to engineering and computer science.* Courier Dover Publications, 2017.

[7] Kapil D Joshi. *Foundations of discrete mathematics.* New Age International, 1989.

7.3 M703: Introduction to Mathematical Modelling

Learning Outcomes: At the end of the course, the students will be able to :

1. Understand the concept of modeling, types of models, scope and limitation.
2. Understand and able to formulate Algebraic Models and their analysis.
3. Understand and able to formulate Discrete Models and their analysis.
4. Understand and able to formulate Continuous Models and their qualitative analysis.
5. Understand and able to explore various bifurcations in the mathematical models.
6. Comprehend Codimension One, Codimension two and Global bifurcations.

Contents:

Unit-I Mathematical Model, types of Mathematical models and properties, Elementary models, Models by nature of environment, Models by the Extent of generality, Principle of modeling, Solution method for models, Characteristics, Advantages and Limitations of a model, Dynamic Models. State variables and parameters, methods and challenges. Model reduction.

Unit-II Algebraic Models: Temperature and the Chirping of a Cricket, Least Squares Fitting of Data, The Global Positioning System, Allometric Models, Dimensional Analysis.

Unit-III Discrete Models: Malthusian Growth Model, Economic Interest Models, Time-Dependent Growth Rate, Qualitative Analysis of Discrete Models, Periodic Points and Bifurcations, Chaos.

Unit-IV Continuous Models: Chemostat, Qualitative Analysis of Continuous Models, A Laser Beam Model, Two Species Competition Model, Predator-Prey Model, Method of Averaging, Linear and Nonlinear Oscillators, Compartmental Models.

Unit-V Bifurcation Theory, Examples and Phase Portraits, Conditions for Bifurcations, Codimension of a Bifurcation, Codimension One Bifurcations in Discrete Systems, Codimension One Bifurcations in Continuous Systems, Global Bifurcations, Symmetry-Breaking Bifurcations.

References:

[1] Antonio Palacios. *Mathematical Modeling: A Dynamical Systems Approach to Analyze Practical Problems in STEM Disciplines*. Springer Nature, 2022.

[2] Joel Kilty and Alex McAllister. *Mathematical Modeling and Applied Calculus*. Oxford University Press, 2018.

[3] Edward A Bender. *An introduction to mathematical modeling*. Courier Corporation, 2000.

7.4 M704: Operations Research

Learning Outcomes: At the end of the course, the students will be able to :

1. Understand the concept of operations research and its scope. Formulate real life problems into linear programming problem and understand the simplex method.
2. Analyze simplex method, duality, sensitivity in linear programming problem.
3. Formulate and solve of linear programming model of game theory.
4. Understand the queuing system. Formulate and solve the queuing theory models.

Contents:

Unit-I Introduction, Nature and Scope of operations research. Linear Programming: Introduction, Mathematical formulation of the problem, Graphical Solution methods, Mathematical solution of linear programming problem, Slack and Surplus variables. Matrix formulation of general linear programming problems.

Unit-II The Simplex Method: Simplex algorithm, Computational procedures, Artificial variables, Two phase Simplex Method, Formulation of linear programming problems and its solution by simplex method.

Unit-III Unrestricted variables, problems of degeneracy, Principle of duality in simplex method, Formation of dual with mixed type of constraints, Solution of primal and dual constraints.

Unit-IV Elementary queuing and inventory models, Steady-state solutions of Markovian queuing models: M/M/1, M/M/1 with limited waiting space, M/M/C, M/M/C with limited waiting space, M/G/1.

Unit-V Game Theory: Introduction, Two persons zero sum games, The maxmin and minimax principles. Graphical Solution: Reduction of game problem to LPP.

References:

[1] Frederick S. Hillier and Gerald J. Lieberman. *Introduction to operations research.* McGraw-Hill Higher Education, 2010.

[2] Kanti Swarup, P. K. Gupta, and Man Mohan. *Operations Research.* Sultan Chand & Sons Publishers, 1977.

[3] JK Sharma. *Operation Research: Theory and Application 4th Edition.* Macmillan Publishers india, 1997.

[4] N Paul Loomba. *Linear programming.* Tata Mcgraw hill publishing company, 1964.

[5] Hamdy A Taha. *Operation Research: An Introduction, 7th.* Prentice Hall-Pearson Education Inc., 2003.

7.5 M705 : Stochastic Analysis

Learning Outcomes: At the end of the course, the students will be able to :

1. Specify a given discrete-time Markov chain in terms of a transition matrix and a transition diagram.
2. Apply first-step analysis to calculate absorption probabilities and mean time to absorption for a discrete-time Markov chain.
3. Understand the Poisson process, in both axiomatic and infinitesimal form.
4. Understand and apply stochastic differential equations.

Contents:

Unit-I Preliminaries: Martingales and properties. Brownian Motion- definition and construction, Markov property, stopping times, strong Markov property zeros of one dimensional Brownian motion.

Unit-II Reection principle, hitting times, higher dimensional Brownian Motion, recurrence and transience,occupation times, exit times, change of time, Levys theorem.

Unit-III Stochastic Calculus: Predictable processes, continuous local martingales, variance and covariance processes.

Unit-IV Integration with respect to bounded martingales and local martingales, Kunita Watanabe inequality, Ito s formula, stochastic integral, change of variables.

Unit-V Stochastic differential equations, weak solutions, Change of measure, Change of time, Girsanovs theorem.

References:

[1] Richard Durrett. *Stochastic calculus: a practical introduction.* CRC press, 2018.

[2] Ioannis Karatzas and Steven Shreve. *Brownian motion and stochastic calculus.* Springer Science & Business Media, 2012.

[3] Bernt Øksendal. *Stochastic differential equations.* Springer, 2003.

[4] J Michael Steele. *Stochastic calculus and financial applications.* Springer, 2001.

8 Semester - VIII

8.1 M801 : Graph Theory

Learning Outcomes: At the end of the course, the students will be able to :

1. Understand the concept of graph, subgraph, walk, path cycles.
2. Understand trees and fundamental circuits and their applications.
3. Understand and analyse planar graph and graph coloring problems.
4. Understand the concept of directed graph and its applications.
5. Be familiar with limitations of graph theory and introduction of complex networks and their representation.

Contents:

Unit-I Introduction to Graphs: Definition of a graph, finite and infinite graphs, incidence of vertices and edges, types of graphs, subgraphs, walks, trails, paths, cycles, connectivity, components of a graph, Eulerian and Hamiltonian graphs, travelling salesman problem, vertex and edge connectivity, matrix representation of graphs, incidence and adjacency matrices of graphs.

Unit-II Trees and Fundamental Circuits: Definition and properties of trees, rooted and binary trees, counting trees, spanning trees, weighted graphs, minimum spanning tree, fundamental circuit, cut set, separability, network flows.

Unit-III Planar Graphs and Graph coloring: Planar graphs, Kuratowski's graphs, detection of planarity, Euler's formula for planar graphs, geometric and combinatorial duals of a planar graphs, coloring of graphs, chromatic numbers, chromatic polynomial, chromatic partitioning, Four color theorem.

Unit-IV Directed Graphs: Types of digraphs, digraphs and binary relations, directed paths and connectedness, Euler digraphs, de Brujin sequences, tournaments.

Unit-V Networks: Networks and their representation, Weighted and directed networks, The adjacency, Laplacian, and incidence matrices, Degree, paths, components, Independent paths, connectivity, and cut sets. Degree centrality, eigenvector centrality, katz centrality, PageRank.

References:

[1] Narsingh Deo. *Graph theory with applications to engineering and computer science.* Courier Dover Publications, 2017.

[2] Md Saidur Rahman et al. *Basic graph theory.* Springer, 2017.

[3] K Erciyes. *Discrete mathematics and graph theory.* Springer, 2021.

8.2 M802 : Advanced Discrete Mathematics

Learning Outcomes: At the end of the course, the students will be able to :

1. Understand Lattice, Boolean algebra as a Lattice.
2. Understand Boolean form, Boolean algebra and Boolean functions and their applications.
3. Understand Grammars and Languages-Phrase-Structure Grammars and its applications.
4. Understand the concept of Finite State Machines.
5. Understand Deterministic, Non-deterministic Finite Automata, Moore and mealy Machines.

Contents:

Unit-I Lattices-Lattices as partially ordered sets. Their properties. Lattices as Algebraic Systems. Sublattices, Direct products, and Homomorphisms. Some Special Lattices e.g., Complete, Complemented and Distributive Lattices. Boolean Algebras-Boolean Algebras as Lattices. Various Boolean Identities. The Switching Algebra example. Subalgebras.

Unit-II Direct Products and Homomorphisms. Join-Irreducible elements, Atoms and Minterms. Boolean Forms and Their Equivalence. Minterm Boolean Forms, Sum of Products Canonical Forms. Minimization of Boolean Functions. Applications of Boolean Algebra to Switching Theory (using AND,OR & NOT gates). The Karnaugh Map Method.

Unit-III Grammars and Languages-Phrase-Structure Grammars. Rewriting Rules. Derivations. Sentential Forms. Language generated by a Grammar. Regular, Context-Free, and Context Sensitive Grammars and Languages. Regular sets, Regular Expressions. Notions of Syntax Analysis, Polish Notations. Conversion of Infix Expressions to Polish Notations. The Reverse Polish Notation.

Unit-IV Introductory Computability Theory-Finite State Machines and their Transition Table Diagrams. Equivalence of finite State Machines. Reduced Machines. Homomorphism.

Unit-V Finite Automata. Acceptors. Non-deterministic Finite Automata and equivalence of its power to that of Deterministic Finite Automata. Moore and mealy Machines. Turing Machine and Partial Recursive Functions. The Pumping Lemma. Kleene's Theorem.

References:

[1] Chung Laung Liu. *Elements of discrete mathematics*. McGraw-Hill, Inc., 1985.

[2] Jean Paul Tremblay and Rampurkar Manohar. *Discrete mathematical structures with applications to computer science*. McGraw-Hill, Inc., 1975.

[3] KLP Mishra and N Chandrasekaran. *Theory of computer science: automata, languages and computation*. PHI Learning Pvt. Ltd., 2006.

[4] Stephen A Wiitala. *Discrete mathematics: a unified approach*. McGraw-Hill, Inc., 1987.

[5] Sriraman Sridharan and Rangaswami Balakrishnan. *Foundations of Discrete Mathematics with Algorithms and Programming*. Chapman and Hall/CRC, 2018.

[6] K Erciyes. *Discrete mathematics and graph theory*. Springer, 2021.

8.3 M803 : Nonlinear Dynamics and Chaos

Learning Outcomes: At the end of the course, the students will be able to :

1. Understand concept and history of Dynamical Systems.
2. Understand the canonical forms, generalized eigenvectors, semi simple – nilpotent decomposition, Jordan canonical form.
3. Understand the one and two dimensional flows and their applications.
4. Understand maps and bifurcations.
5. Understand the Lorenz equations, a chaotic waterwheel and properties of the Lorenz equations.

Contents:

Unit-I: Introduction to Dynamical Systems, history of dynamics, phase portraits, vector fields, nullclines, flows, discrete dynamical systems, 1-d maps. Fixed points, linearization of vector fields, canonical forms, generalized eigenvectors, semisimple – nilpotent decomposition, Jordan canonical form.

Unit-II One dimensional flows: fixed points and stability, population growth model, linear stability analysis, existence and uniqueness of solution, impossibility of oscillations.
Introduction to bifurcation, Saddle-Node bifurcation, Transcritical bifurcation, Pitchfork bifurcation, imperfect bifurcations and catastrophes.

Unit-III Two dimensional flows: Linear systems, classifications of linear systems, phase plane, phase portraits, existence, uniqueness and topological consequences, fixed points and linearization. Conservative systems and reversible systems, index theory.

Unit-IV Limit cycles, ruling out closed orbit, Poincare-Bendixson theorem, Lienard systems. Relaxation oscillations, weakly nonlinear oscillators. Bifurcations: Saddle-Node, Transcritical and Pitchfork bifurcations, Hopf bifurcations, Global bifurcations of cycles.

Unit-V Hysteresis, coupled oscillators and quasiperiodicity, Poincare maps. Chaotic dynamics: Lorenz equations, a chaotic waterwheel, properties of the Lorenz equations.

References:

[1] Steven H Strogatz. *Nonlinear dynamics and chaos: with applications to physics, biology, chemistry, and engineering.* CRC press, 2018.

[2] Stephen Lynch. *Dynamical systems with applications using MATLAB.* Springer, 2004.

[3] JA Rial. *Chaos: An Introduction to Dynamical Systems.* Sigma XI-The Scientific Research Society, 1997.

[4] Morris W Hirsch, Stephen Smale, and Robert L Devaney. *Differential equations, dynamical systems, and an introduction to chaos.* Academic press, 2012.

[5] Stephen Lynch. *Dynamical systems with applications using Mathematica.* Springer, 2007.

[6] Kathleen T Alligood, Tim D Sauer, James A Yorke, and David Chillingworth. *Chaos: an introduction to dynamical systems.* Philadelphia, Society for Industrial and Applied Mathematics., 1998.

8.4 M804: Mathematical Biology

Learning Outcomes: At the end of the course, the students will be able to :

1. Understand the Simple Single Species Models, Harvesting in Two-species Models and their applications.
2. Understand the Continuous Single-Species Population Models with Delays.
3. Develop the ability to explain mathematical results in language understandable by biologists.
4. Understand and apply the concept of stability of a fixed point solution of a system of ordinary differential equations.
5. Solve mathematically and interpret biologically simple problems involving one- and two-species ecosystems, epidemics and biochemical reactions.
6. Demonstrate ability to analysis qualitative aspects of ODEs in a biological modelling context.
7. Apply appropriate techniques to solve a given model of a biological problem.
8. Select appropriate approaches/methods and tools to generate mathematical models of aspects of biology.
9. Be able to formulate and critically evaluate epidemiological models.
10. Critically evaluate the merits and weaknesses of biological models.

Contents:

Unit-I Simple Single Species Models: Continuous Population Models, Exponential Growth, The Logistic Population Model, Harvesting in Population Models, Constant-Yield and Constant-Effort Harvesting, Eutrophication of a Lake: A Case Study.
Discrete–Time Metered Models, Systems of Two Difference Equations, Oscillation in Flour Beetle Populations: A Case Study.

Unit-II Continuous Single-Species Population Models with Delays: Models with Delay in Per Capita Growth Rates, Delayed-Recruitment Models, Models with Distributed Delay, Harvesting in Delayed Recruitment Models, Nicholson's Blowflies: A Case Study.

Unit-III Models for Interacting Species: The Lotka–Volterra Equations, The Chemostat Model, Equilibria and Linearization, Qualitative Behavior of Solutions of Linear Systems, Periodic Solutions and Limit Cycles, Species in Competition, Kolmogorov Models, Mutualism, The Spruce Budworm: A Case Study.
The Community Matrix, the Nature of Interactions Between Species, Invading Species and Coexistence.

Unit-IV Harvesting in Two-species Models: Harvesting of Species in Competition, Harvesting of Predator–Prey Systems, Intermittent Harvesting of Predator–Prey Systems, Some Economic Aspects of Harvesting, Optimization of Harvesting Returns, A Nonlinear Optimization Problem, Economic Interpretation of the Maximum Principle.

Unit-V Models for Populations with Age and Spatial Structure: Linear model with age structure, The Method of Characteristics, Nonlinear Continuous Models, Models with Discrete Age Groups, Some Simple Examples of Metapopulation Models, A General Metapopulation Model, A Metapopulation Model with Residence and Travel, The Diffusion Equation, Solution by Separation of Variables, Solutions in Unbounded Regions, Linear Reaction–Diffusion Equations, Nonlinear Reaction–Diffusion Equations, Diffusion in Two Dimensions.

References:

[1] Fred Brauer, Carlos Castillo-Chavez, and Carlos Castillo-Chavez. *Mathematical models in population biology and epidemiology.* Springer, 2012.

[2] Mark Kot. *Elements of mathematical ecology.* Cambridge University Press, 2001.

[3] James D Murray. *Mathematical biology: I. An introduction. Interdisciplinary applied mathematics.* Springer, 2002.

[4] James D Murray. *Mathematical biology II: spatial models and biomedical applications.* Springer New York, 2001.

8.5 M805 : Computational Mathematics III

Learning Outcomes: At the end of the course, the students will be able to :

1. Understand the programming language SAGE and its uses.
2. Plot the graphs of 2d and 3d objects in various forms.
3. Understand the use of SAGE to explore calculus of single and multi-variables.
4. Understand the use of SAGE to explore concepts in Group-Theory, Number-Theory and Combinatorics.

Contents:

Unit-I Introduction to SAGE, using SAGE as an advanced calculator

Unit-II Plotting graphs of 2d and 3d objects in various forms

Unit-III Use of SAGE to explore calculus of single and multi-variables.

Unit-IV Use of SAGE to explore row transformations, linear transformations, Gram-Schmidt process, application of matrix diagonalization, matrix factorizations with applications to least square problems and image processing etc.

Unit-V Use of SAGE to explore concepts in Group-Theory, Number-Theory and Combinatorics.

References:

[1] Sang-gu lee and ajit kumar. linear algebra with sage, free online available at. `http://matrix.skku.ac.kr/2015-Album/Big-Book-LinearAlgebra-Eng-2015.pdf`.

[2] George A Anastassiou and Razvan A Mezei. *Numerical analysis using sage.* Springer, 2015.

9 Semester - X

9.1 ME01: Dynamical Systems Using Matlab

Learning Outcomes: At the end of the course, the students will be able to :

1. Understand basics of Matlab Software.
2. Explain how ordinary differential equations (ODEs) give rise to dynamical systems.
3. Define the state space, its limit sets and attractors.
4. Explain how the state space dimension limits the possible dynamics.
5. Sketch the limit set and, starting from this, characterize the main features in the flow of a dynamical system given by ODEs in the plane.
6. Explain the concept of chaos in dynamical systems and state some properties of a chaotic dynamical system

Contents:

Unit-I **Introduction to Matlab:** Arithmetic Operations, built-in-MATH functions, scalar variables, creating arrays, built-in functions for handling arrays, mathematical operations with arrays, script files, two dimensional plots, programming in MATLAB, polynomial, curve fitting, and interpolation, three-dimensional plots.

Unit-II Discrete Dynamical Systems: One-dimensional maps, cobweb plot: graphical representation of an orbit, stability of fixed points, periodic points, the family of logistic maps, sensitive dependence on initial conditions, analysis of logistic map, Periodic Windows, Feigenbaum number, chaos in logistic map.

Unit-III Higher-dimensional maps, sinks, sources, and saddles, nonlinear maps and the jacobian matrix, stable and unstable manifolds, lyapunov exponents, Numerical Calculation of Lyapunov Exponent, chaotic orbits. Strange Attractors, Gaussian and Hénon Maps. Julia Sets and the Mandelbrot Set.

Unit-IV Differential Dynamical Systems: Differential dynamical systems, existence and uniqueness theorem, phase portraits, vector fields, nullclines, flows, fixed points, linearization of vector fields, planar systems, canonical forms, eigenvectors defining stable and unstable manifolds, phase portraits of linear systems in the plane, linearization and Hartman's theorem, limit cycles, existence and uniqueness of limit cycles in the plane, Lyapunov functions and stability.

Unit-V Nonlinear systems and stability, bifurcations of nonlinear systems, normal forms, multistability and bistability, the Rössler system and chaos, the Lorenz equations, Chua's circuit, and the Belousov–Zhabotinski reaction.

References:

[1] Stephen Lynch. *Dynamical systems with applications using MATLAB.* Springer, 2004.

[2] Steven H Strogatz. *Nonlinear dynamics and chaos: with applications to physics, biology, chemistry, and engineering.* CRC press, 2018.

[3] Morris W Hirsch, Stephen Smale, and Robert L Devaney. *Differential equations, dynamical systems, and an introduction to chaos.* Academic press, 2012.

[4] Stephen Lynch. *Dynamical systems with applications using Mathematica.* Springer, 2007.

[5] Kathleen T Alligood, Tim D Sauer, James A Yorke, and David Chillingworth. *Chaos: an introduction to dynamical systems.* Philadelphia, Society for Industrial and Applied Mathematics., 1998.

9.2 ME02: Commutative Algebra

Learning Outcomes: At the end of the course, the students will be able to :

1. Knows basic definitions concerning elements in rings, classes of rings, and ideals in commutative rings.
2. Know constructions like tensor product and localization, and the basic theory for this.
3. Know basic theory for noetherian rings and Hilbert basis theorem.
4. Know basic theory for integral dependence, and the Noether normalization lemma.
5. Have insight in the correspondence between ideals in polynomial rings, and the corresponding geometric objects: affine varieties.
6. Know basic theory for support and associated prime ideals of modules, and know primary decomposition of ideals in noetherian rings.
7. Know the theory of Gröbner bases and Buchbergers algorithm.
8. Know the theory of Hilbert series and Hilbert polynomials.
9. Know dimension theory of local rings.

Contents:

Unit-I Prime and maximal ideals in a commutative ring, nil and Jacobson radicals, Nakayamas lemma, local rings

Unit-II Rings and modules of fractions, correspondence between prime ideals, localization.

Unit-III Modules of finite length, Noetherian and Artinian modules. Primary decomposition in a Noetherian module, associated primes, support of a module.

Unit-IV Graded rings and modules, Artin-Rees, Krull-intersection, Hilbert-Samuel function of a local ring, dimension theory, principal ideal theorem.

Unit-V Integral extensions, Noethers normalization lemma, Hilberts Nullstellensatz (algebraic and geometric versions).

References:

[1] W Jonsson. *Introduction to Commutative Algebra.* Cambridge University Press, 1970.

[2] David Eisenbud. *Commutative algebra: with a view toward algebraic geometry.* Springer Science & Business Media, 2013.

[3] Hideyuki Matsumura. *Commutative ring theory.* Cambridge university press, 1989.

[4] Srinivasacharya Raghavan, Balwant Singh, and Ramaiyengar Sridharan. *Homological methods in commutative algebra.* Oxford University Press, 1975.

[5] Balwant Singh. *Basic commutative algebra.* World Scientific Publishing Company, 2011.

9.3 ME03: Financial Mathematics

Learning Outcomes: At the end of the course, the students will be able to :

1. Describe various financial instruments.
2. Explain how probability enters modern descriptions of financial instruments.
3. Explain the time-value of money and use it to analyse simple cashflow models.
4. Explain the notion of arbitrage and apply it to determine the fair price of various financial instruments.
5. Appreciate that even the most basic of financial models requires a profound combination of techniques from various branches of mathematics.
6. Calculate the no-arbitrage price of a European option using the Black-Scholes model.
7. Explain several models appearing in financial mathematics, particularly those which are an extension or modification of geometric Brownian motion.
8. Describe how techniques developed in differential equations and probability are applied to analysis of various financial models.
9. Explain why comprehension of descriptive definitions associated with financial models is essential.

Contents:

Unit-I Review Of probability, finite probability space.

Unit-II Derivatives security, interest rates, other financial instruments, Arbitrage and pricing, risk less issue, yield curves, mean terms matching and immunization, interest rate models.

Unit-III Dependent annual rates of return, random walk and Markov process, stochastic calculus.

Unit-IV option pricing, portfolio optimization, Fokker-plank equation, distribution and green functions.

Unit-V Feynman-kac formula options, dividends revisited. Exotic options.

References:

[1] Richard F Bass. The basics of financial mathematics. *Department of Mathematics, University of Connecticut*, 2003.

[2] Paul Wilmott, Susan Howson, Sam Howison, Jeff Dewynne, et al. *The mathematics of financial derivatives: a student introduction*. Cambridge university press, 1995.

[3] C. Gardiner. *Stochastic Methods: A Handbook for the Natural and Social Sciences*. Springer Series in Synergetics. Springer Berlin Heidelberg, 2010.

[4] John C Hull. *Options futures and other derivatives*. Pearson Education India, 2003.

9.4 ME04: Nonlinear Analysis

Learning Outcomes: At the end of the course, the students will be able to :

1. Understand calculus in Banach Space.
2. Understand Fundamentals of Monotone operators.
3. Understand Fixed point theorems.
4. Understand applications of Monotone Operators and Fixed point theory.

Contents:

Unit-I Calculus in Banach space:
Various form of continiuty, geometry in normed spaces and duality mappings, Gateaux and Frechet derivative, properties of derivatives, Taylor theorem, inverse function theorem and impliocit function theorem, subdifferential of convex function.

Unit-II Monotone operators:
Monotone operators, Maximal monotone operators and its properties, constructive solution of operator equations, subdifferential and monotonicity, some generalization of monotone operator.

Unit-III Fixed point theorems: Banach contraction principle and its generalizations, nonexpansive mappings, fixed point theorem of Brouwder and Schauder. Fixed point theorems for multi-functions, common fixed point theorems, sequence of contractions, generalized contractions and fixed points.

Unit-IV Applications of monotone operators theory:
Introduction, Sobolev space, differential equation, nonlinear differential equations, integral equation, Nonlinear ammerstein integral equation, Generalized Hammerstein integral equation.

Unit-V Unit-V: Applications of fixed point theorems:
Application to Geometry of Banach Spaces, Application to System of Linear Equations, Perron–Frobenius, Fundamental Theorem of Algebra, Game Theory and Nash Equilibria, Differential equations, integral equations.

References:

[1] Mohan C Joshi and Ramendra K Bose. *Some topics in nonlinear functional analysis.* John Wiley & Sons, 1985.

[2] Hemant Kumar Pathak. *An introduction to nonlinear analysis and fixed point theory.* Springer, 2018.

[3] Eberhard Zeidler and Peter R Wadsack. *Nonlinear Functional Analysis and Its Applications: Fixed-point Theorems.* Springer-Verlag, 1993.

[4] Rajendra Akerkar. *Nonlinear functional analysis.* Alpha Science International, Limited, 1999.

9.5 ME05: Differential Topology

Learning Outcomes: At the end of the course, the students will be able to :

1. Give an account of central concepts and definitions in differential topology.
2. State Sard's theorem and some of its applications.
3. Define and compute mapping degree and intersection number of two submanifolds.
4. Define index of a vector field and state the Poincaré-Hopf theorem.
5. Define Morse function and outline a proof of existence.
6. State the classification of one- and two-dimensional manifolds.

Contents:

Unit-I Differentiable functions on R^n, Review of Differentiable functions f : R^n to R^m, implicit and invers function theorems, immersions and Submersions, critical points, critical and regular values.

Unit-II Manifolds: Level sets, sub-manifolds of R^n, immersed and embedded sub-manifolds, tangent spaces, differentiable functions between sub-manifolds of R^n, abstract differential manifolds and tangent spaces.

Unit-III Differentiable functions on Manifolds: Differentiable functions f : M N, critical points, Sard's theorem, non-degenerate critical points, Morse Lemma, Manifolds with boundary, Brouwer fixed point theorem, mod 2 degree of a mapping

Unit-IV Transversality: Orientation of Manifolds, oriented intersection number, Brouwer degree, transverse intersections.

Unit-V Integration on Manifolds: Vector field and Differential forms, integration of forms, Stokes' theorem, exact and closed forms, Poincar Lemma, Introduction to de Rham theory.

References:

[1] John Milnor and David W Weaver. *Topology from the differentiable viewpoint.* Princeton university press, 1997.

[2] Victor Guillemin and Alan Pollack. *Differential topology.* American Mathematical Soc., 2010.

[3] Morris W Hirsch. *Differential topology.* Springer Science & Business Media, 2012.

9.6 ME06: Introduction to Cryptography

Learning Outcomes: At the end of the course, the students will be able to :

1. Describe basic concepts of cryptography and steganography; define plaintext, ciphertext and key.
2. Describe substitution and other traditional ciphers and perform encryption/decryption with them.
3. Describe stream ciphers including the Vigenère cipher, one-time pad and shift registers, explain their advantages and disadvantages, and perform encryption/decryption with them for simple situations.
4. Explain simple concepts associated with security of ciphers, including statistical attack methods; calculate the number of possible keys for simple substitution and stream ciphers.
5. Define Euler's function and Carmichael's function; calculate them for small integers.
6. Define a Latin square and perform actions on it such as finding the transpose or adjugate.
7. State and prove Shannon's Theorem (for one-time pads).
8. Describe the basic principles of public-key cryptography including complexity issues and knapsack, RSA and El-Gamal ciphers.
9. Describe the basic principles of digital signatures and authentication; secret sharing

Contents:

Unit-I Classical Cryptosystems: Some Simple Cryptosystems, Monoalphabatic and Polyalphabatic cipher, The Shift Cipher, The Substitution Cipher, The Affine Cipher, The Vigenere Cipher, The Hill Cipher, The Permutation Cipher, Cryptanalysis, Some Cryptanalytic Attacks, Stream ciphers, Synchronous Stream Cipher, Linear Feedback Shift Register (LFSR), Non-Synchronous stream Cipher, Autokey Cipher.

Unit-II Block Ciphers: Mode of operations in block cipher: Electronic Codebook (ECB), Ciphertext Chaining (CBC), Ciphertext FeedBack (CFB), Output FeedBack (OFB), Counter (CTR).
DES & AES: The Data Encryption Standard (DES), Feistel Ciphers, Description of DES, Security analysis of DES, Differential & Linear Cryptanalysis of DES, Triple DES, The Advanced Encryption Standard (AES), Finite field GF(28), Description of AES, analysis of AES.

Unit-III Shannon's Theory of Perfect Secrecy: Perfect Secrecy, Birthday Paradox, Vernam One Time Pad, Random Numbers, Pseudorandom Numbers. Prime Number Generation: Trial Division, Fermat Test, Carmichael Numbers, Miller Rabin Test, Random Primes.

Unit-IV Public Key Cryptography: Principle of Public Key Cryptography, RSA Cryptosystem, Factoring problem, Cryptanalysis of RSA, RSA-OAEP, Rabin Cryptosystem, Security of Rabin Cryptosystem, Quadratic Residue Problem, Diffie-Hellman (DH) Key Exchange Protocol, Discrete Logarithm Problem (DLP), ElGamal Cryptosystem, ElGamal & DH, Algorithms for DLP. Elliptic Curve, Elliptic Curve Cryptosystem (ECC), Elliptic Curve Discrete Logarithm Problem (ECDLP).

Unit-V Cryptographic Hash Functions: Hash and Compression Functions, Security of Hash Functions, Modification Detection Code (MDC0. Message Authentication Codes (MAC), Random Oracle Model, Iterated Hash Functions, Merkle-Damgard Hash Function, MD-5, SHA-1, Others Hash Functions. **Digital Signatures:** Security Requirements for Signature Schemes, Signature and Hash Functions, RSA Signature, ElGamal Signature, Digital Signature Algorithm (DSA), ECDSA, Undeniable Signature, Blind Signature.

References:

[1] Johannes Buchmann. *Introduction to cryptography.* Springer, 2004.

[2] Sahadeo Padhye, Rajeev A Sahu, and Vishal Saraswat. *Introduction to cryptography.* CRC Press, 2018.

[3] Douglas R Stinson. *Cryptography: theory and practice.* Chapman and Hall/CRC, 2005.

[4] Bruce Schneier. *Applied cryptography: protocols, algorithms, and source code in C.* john wiley & sons, 2007.

[5] Debdeep Mukhopadhyay and BA Forouzan. Cryptography and network security. *Noida: Tata Mcgraw Hill,* 2011.

[6] Wenbo Mao. *Modern cryptography: theory and practice.* Pearson Education India, 2003.

[7] William Stallings. *Cryptography and network security.* Pearson Education India, 2006.

9.7 ME07: Introduction to Nonlinear Optimization

Learning Outcomes: At the end of the course, the students will be able to :

1. Demonstrate knowledge and understanding of nonlinear programming modelling techniques.
2. Demonstrate knowledge and understanding of nonlinear programming solution algorithms.
3. Understand global and local optima.
4. Understand Least Square, Newton Method in optimization.
5. Understand convex function and its role in optimization.

Contents:

Unit-I Mathematical Preliminaries, the Space R^n, $R^{n\times m}$, Inner Products and Norms, Eigen values and Eigen vectors, Basic Topological Concepts.

Unit-II Optimality Conditions for Unconstrained Optimization: Global and Local Optima, Classification of Matrices, Second Order Optimality Conditions, Global Optimality Conditions, Quadratic Functions.

Unit-III Least Squares: Solution of over determined Systems, Data Fitting, Regularized Least Squares, Denoising, Nonlinear Least Squares. Descent Directions Methods, The Gradient Method, The Condition Number, Diagonal Scaling, The Gauss–Newton Method, The Fermat–Weber Problem, Convergence Analysis of the Gradient Method.

Unit-IV Newton's Method, Pure Newton's Method, Damped Newton's Method, The Cholesky Factorization. Convex Sets, Algebraic Operations with Convex Sets, The Convex Hull, Convex Cones, Topological Properties of Convex Sets, Extreme Points.

Unit-V Convex Functions, First Order Characterizations of Convex Functions, Second Order Characterization of Convex Functions, Operations Preserving Convexity, Level Sets of Convex Functions, Maxima of Convex Functions, Convexity and Inequalities, Convex Optimization, The Orthogonal Projection Operator, Optimization over a Convex Set, Stationarity in Convex Problems, The Orthogonal Projection Revisited, The Gradient Projection Method, Sparsity Constrained Problems.

References:

[1] Amir Beck. *Introduction to nonlinear optimization: Theory, algorithms, and applications with MATLAB*. SIAM, 2014

[2] Wenyu Sun and Ya-Xiang Yuan. *Optimization theory and methods: nonlinear programming*, volume 1. Springer Science & Business Media, 2006.

[3] Francisco J Aragón, Miguel A Goberna, Marco A López, and Margarita ML Rodríguez. *Nonlinear optimization*. Springer, 2019.

[4] HA Eiselt and Carl-Louis Sandblom. *Nonlinear optimization*. Springer, 2019.

9.8 ME08: Complex Network

Learning Outcomes: At the end of the course, the students will be able to :

1. State the basic metrics to characterise the structure of a network.
2. Calculate various centrality measures.
3. Derive the degree distribution and the characteristic path length of a random graph.
4. Explain what is meant by "small-world" and "scale-free" networks.
5. Derive the degree distribution of the Barabasi-Albert model.
6. State how to describe degree correlations in a network.
7. State methods to extract motifs and community structures in networks.
8. Describe the basic features of various real social, biological and man-made networks.

Contents:

Unit-I Fundamentals of Graph Theory, Directed, Weighted and Bipartite Graphs, Trees. Complex Network, Basics, history and importance of Complex Network.

Unit-II Centrality Measures: The Importance of Being Central, Connected Graphs and Irreducible Matrices, Degree and Eigenvector Centrality, Measures Based on Shortest Paths, Group Centrality.

Unit-III Random Graphs: Erd˝os and Rényi (ER) Models, Degree Distribution, Trees, Cycles and Complete Sub-graphs, Giant Connected Component, Scientific Collaboration Networks, Characteristic Path Length.

Unit-IV Small-World Networks: Six Degrees of Separation, The Brain of a Worm, Clustering Coefficient, The Watts–Strogatz (WS) Model, Variations to the Theme, Navigating Small-World Networks.

Unit-V Generalised Random Graphs: The World Wide Web, Power-Law Degree Distributions, The Configuration Model, Random Graphs with Arbitrary Degree Distribution, Scale-Free Random Graphs, Probability Generating Functions. Models of Growing Graphs, Degree Correlations.

References:

[1] Guido Caldarelli. *Complex Networks: Principles, Methods and Applications*. Oxford University Press, 2018.

[2] Maarten Van Steen. *An introduction to Graph theory and complex networks*. Van Steen, Maarten, 2010.

[3] S Dorogovtsev. *Complex networks*. Oxford University Press Oxford, 2010.

[4] Ernesto Estrada. *The structure of complex networks: theory and applications*. Oxford University Press, 2012.

9.9 ME09: Representation Theory of Finite Groups

Learning Outcomes: At the end of the course, the students will be able to :

1. Know in particular simple modules and semi-simple algebras and they will be familiar with examples.
2. appreciate important results in the course such as the Jordan-Hölder Theorem, Schur's Lemma, and the Wedderburn Theorem.
3. Be familiar with the classification of semi-simple algebras over C and be able to apply this to representations and characters of finite groups.

Contents:

Unit-I Recollection of left and right modules, direct sums, tensor products

Unit-II Semi-simplicity of rings and modules, Schur's lemma, Maschk's Theorem

Unit-III Wedderburn's Structure Theorem. The group algebra.

Unit-IV Representations of a finite group over a field, induced representations, characters, orthogonality relations

Unit-V Representations of some special groups. Burnside's pa qb theorem.

References:

[1] Michael Artin and William F Schelter. *Graded algebras of global dimension 3.* Academic Press, 1987.

[2] Martin Burrow. *Representation theory of finite groups.* Courier Corporation, 2014.

[3] David Steven Dummit and Richard M Foote. *Abstract Algebra.* Wiley Hoboken, 2004.

[4] Nathan Jacobson. *Lectures in Abstract Algebra: II. Linear Algebra.* Springer Science & Business Media, 2013.

[5] Serge Lang. *Algebra.* Springer Science & Business Media, 2012.

[6] Jean-Pierre Serre et al. *Linear representations of finite groups.* Springer, 1977.

9.10 ME10: Algebraic Number Theory

Learning Outcomes: At the end of the course, the students will be able to understand:

1. field extensions, minimum polynomial, algebraic numbers, conjugates, discriminants, Gaussian integers, algebraic integers, integral basis examples: quadratic fields.
2. norm of an algebraic number, existence of factorisation.
3. relationship between factorisation of number and of ideals.
4. Noetherian rings, Rings of dimension one.
5. Cyclotomic extensions and calculation of the discriminant.
6. statement of Minkowski convex body theorem.
7. Ideal class group, statement of Dirichlet's theorem on finite generation of units.

Contents:

Unit-I Field extensions and examples of field extensions of rational numbers, real numbers and complex numbers. Monic polynomials, Integral extensions, Minimal polynomial, Characteristic polynomial.

Unit-II Integral closure and examples of rings which are integrally closed. Examples of rings which are not integrally closed. The ring of integers. The ring of Gaussian integers. Quadratic extensions and description of the ring of integers in quadratic number fields. Units in quadratic number fields and relations to continued fractions.

Unit-III Noetherian rings, Rings of dimension one. Dedekind domains. Norms and traces. Derive formulae relating norms and traces for towers of field extensions. Discriminant and calculations of the discriminant in the special context of quadratic number fields. Different and it's applications.

Unit-IV Cyclotomic extensions and calculation of the discriminant in this case. Factorization of ideals into prime ideals and its relation to the discriminant. Ramification theory, residual degree and its relation to the degree of the extension. Ramified primes in quadratic number fields.

Unit-V Ideal class group. Geometric ideas involving volumes. Minkowski's theorem and its application to proving finiteness of the ideal class group. Real and complex embeddings. Structure of finitely generated abelian groups. Dirichlet's Unit Theorem and the rank of the group of units. Discrete valuation rings, Local fields.

References:

[1] Gerald J Janusz. *Algebraic number fields.* American Mathematical Soc., 1996.

[2] Jürgen Neukirch. *Algebraic number theory.* Springer Science & Business Media, 2013.

[3] Daniel A Marcus and Emanuele Sacco. *Number fields.* Springer, 1977.

9.11 ME11: Algebraic Topology

Learning Outcomes: At the end of the course, the students will be able to :

1. Explain the fundamental concepts of algebraic topology and their role in modern mathematics and applied contexts.
2. Demonstrate accurate and efficient use of algebraic topology techniques.
3. Demonstrate capacity for mathematical reasoning through analyzing, proving and explaining concepts from algebraic topology.
4. Apply problem-solving using algebraic topology techniques applied to diverse situations in physics, engineering and other mathematical contexts.
5. Understand the concept of Paths, Homotopies of paths, covering space, orbit space, free groups.

Contents:

Unit-I Review of quotient spaces and its universal properties. Examples on $\mathbf{RP}^n$, Klein's bottle, Mobius band, $\mathbf{CP}^n$, $SO(n, R)$. Connectedness and path connectedness of spaces such as $SO(n, R)$ and other similar examples. Topological groups and their basic properties. Proof that if H is a connected subgroup such that G/H is also connected (as a topological space) then G is connected. Quaternions, $\mathbf{S}^3$ and $SO(3, R)$. Connected, locally path connected space is path connected.

Unit-II Paths and homotopies of paths. The fundamental group and its basic properties. The fundamental group of a topological group is abelian. Homotopy of maps, retraction and deformation retraction. The fundamental group of a product. The fundamental group of the circle. Brouwer's fixed poin theorem. Degree of a map. Applications such as the fundamental theorem of algebra, Borsuk-Ulam theorem and the Perron Frobenius theorem.

Unit-III Covering spaces and its basic properties. Examples such as the real line as a covering space of a circle, the double cover $\eta : \mathbf{S}^n \to \mathbf{RP}^n$, the double cover $\eta : \mathbf{S}^3 \to SO(3,R)$. Relationship to the fundamental group. Lifting criterion and Deck transformations. Equivalence of covering spaces. Universal covering spaces. Regular coverings and its various equivalent formulations such as the transitivity of the action of the Deck group. The Galois theory of covering spaces.

Unit-IV Orbit spaces. Fundamental group of the Klein's bottle and torus. Relation between covering spaces and Orientation of smooth manifolds. Non orientabilty of $\mathbf{RP}^2$ illustrated via covering spaces.

Unit-V Free groups and its basic properties, free products with amalgamations. Concept of push outs in the context of topological spaces and groups. Seifert Van Kampen theorem and its applications. Basic notions of knot theory such as the group of a knot. Wirtinger's algorithm for calculating the Group of a knot illustrated with simple examples.

References:

[1] Elon Lages Lima. *Fundamental groups and covering spaces*. AK Peters/CRC Press, 2003.

[2] WS Massey. *Introduction to algebraic topology*. Springer Verlag, 1967.

9.12 ME12: Differential Geometry & Applications

Learning Outcomes: At the end of the course, the students will be able to :

1. Precisely define and describe basic geometric concepts, such as curves and surfaces.
2. Explain the main ideas of Gauss' formula, Principal Curvatures, Euler's theorem.
3. Understand the concept of Curvature of curves in E^n .
4. Understand the concept of Euler's Theory of curves on Surfaces.
5. Understand the concept of Gauss's theory of Curvature of Surfaces.
6. Understand the concept of More Surface theory.
7. Understand the concept of Modern perspective on surfaces, Tangent plane.

Contents:

Unit-I Curvature of curves in E^n : Parametrized Curves, Existence of Arc length parametrization, Curvature of plane curves, Frennet-Serret theory of (arc-length parametrized) curves in E^3, Curvature of (arc-length parametrized) curves in En, Curvature theory for parametrized curves in E^n. Significance of the sign of curvature, Rigidity of curves in E^n.

Unit-II Euler's Theory of curves on Surfaces : Surface patches and local coordinates, Examples of surfaces in E^3, curves on a surface, tangents to the surface at a point, Vector fields along curves, Parallel vector fields, vector fields on surfaces, Normal vector fields, the First Fundamental form, Normal curvature of curves on a surface, Geodesics, geodesic Curvature, Christoffel symbols, Gauss' formula,Principal Curvatures, Euler's theorem.

Unit-III Gauss' theory of Curvature of Surfaces : The Second Fundamental Form, Weingarten map and the Shape operator, Gaussian Curvature, Gauss' Theorema Egregium, Guass-Codazzi equations,Computation of First/Second fundamental form, curvature etc. for surfaces of revolution and other examples.

Unit-IV More Surface theory: Isoperimetric Inequality, Mean Curvature and Minimal Surfaces (introduction), surfaces of constant curvature, Geodesic coordinates, Notion of orientation, examples of non-orientable surfaces, Euler characteristic, statement of Gauss-Bonnet Theorem.

Unit-v Modern perspective on surfaces, Tangent planes, Parallel transport, Afine connection, Reimannian metrics on surfaces.

References:

[1] Andrew N Pressley. *Elementary differential geometry.* Springer Science & Business Media, 2010.

[2] John A Thorpe. *Elementary topics in differential geometry.* Springer Science & Business Media, 2012.

[3] Manfredo P Do Carmo. *Differential geometry of curves and surfaces: revised and updated second edition.* Courier Dover Publications, 2016.

[4] Richard S Millman and George D Parker. *Elements of differential geometry.* Prentice Hall, 1977.

9.13 ME13: Fuzzy Set Theory & Its Applications

Learning Outcomes: At the end of the course, the students will be able to :

1. Understand the various concept in fuzzy sets.
2. Understand the extension principle and operations on fuzzy sets.
3. Understand the fuzzy relations on Fuzzy sets.
4. Understand the fuzzy equivalence relations and relational equations.
5. Explain fuzzy measure and possibility theory.

Contents:

Unit-I Fuzzy sets-Basic definitions, α-level sets. Convex fuzzy sets. Basic operations on fuzzy sets. Types of fuzzy sets. Cartesian products, Algebraic products. Bounded sum and difference, t-norms and t-conorms.

Unit-II The Extension Principle- The Zadeh's extension principle. Image and inverse image of fuzzy sets. Fuzzy numbers. Elements of fuzzy arithmetic.

Unit-III Fuzzy Relations on Fuzzy sets, Composition of Fuzzy relations. Min-Max composition and its properties.

Unit-IV Fuzzy equivalence relations. Fuzzy compatibility relations. Fuzzy relation equations. Fuzzy graphs, Similarity relation.

Unit-V Possibility Theory-Fuzzy measures. Evidence theory. Necessity measure. Possibility measure. Possibility distribution. Possibility theory and fuzzy sets. Possibility theory versus probability theory.

References:

[1] Hans-Jürgen Zimmermann. *Fuzzy set theory—and its applications.* Springer Science & Business Media, 2011.

[2] George Klir and Bo Yuan. *Fuzzy sets and fuzzy logic.* Prentice hall New Jersey, 1995.

[3] M Ganesh. *Introduction to fuzzy sets and fuzzy logic.* PHI Learning Pvt. Ltd., 2006.

[4] James J Buckley and Esfandiar Eslami. *An introduction to fuzzy logic and fuzzy sets.* Springer Science & Business Media, 2002.

[5] Kazuo Tanaka and Kazuo Tanaka. *An introduction to fuzzy logic for practical applications.* Springer, 1997.

9.14 ME14: Wavelets

Learning Outcomes: At the end of the course, the students will be able to :

1. Understand the basic concept of wavelet theory and ways of constructing wavelets.
2. Understand and apply unitary folding operators and the smooth projections.
3. Understand the concept of multi-resolution analysis and construction of compactly supported wavelets.
4. Understand the characterization of Lemarie-Meyer wavelets, Franklin wavelets and spline wavelets on the real line.
5. Understand and apply decomposition and reconstruction algorithms for wavelets.

Contents:

Unit-I Preliminaries-Different ways of constructing wavelets- Orthonormal bases generated by a single function: the Balian-Low theorem. Smooth projections on $L^2(\mathbb{R})$.

Unit-II Local sine and cosine bases and the construction of some wavelets. The unitary folding operators and the smooth projections.

Unit-III Multiresolution analysis and construction of wavelets. Construction of compactly supported wavelets and estimates for its smoothness. Band limited wavelets.

Unit-IV Orthonormality. Completeness. Characterization of Lemarie-Meyer wavelets and some other characterizations. Franklin wavelets and Spline wavelets on the real line.

Unit-V Orthonormal bases of piecewise linear continuous functions for $L^2(\mathbb{T})$. Orthonormal bases of periodic splines. Periodization of wavelets defined on the real line.

References:

[1] Albert Boggess and Francis J Narcowich. *A first course in wavelets with Fourier analysis*. John Wiley & Sons, 2015.

[2] Eugenio Hernández and Guido Weiss. *A first course on wavelets*. CRC press, 1996.

[3] Przemyslaw Wojtaszczyk. *A mathematical introduction to wavelets*. Cambridge University Press, 1997.

[4] David F Walnut. *An introduction to wavelet analysis*. Springer Science & Business Media, 2002.

[5] Gerald Kaiser and Lonnie H Hudgins. *A friendly guide to wavelets*. Springer, 1994.

9.15 ME15: Mathematical Methods

Learning Outcomes: At the end of the course, the students will be able to :

1. Understand Integral equations, its types and properties.
2. Understand different kernals, Eigen values and functions of Integral operator.
3. Solve Fredholm Integral Equations of the second kinds.
4. Understand Calculus of variations, Euler - Lagrange equations.
5. Understand Variational formulation of boundary value problems.

Contents:

Unit-I Integral equations, Introduction, Abel,s Problem, Fredholm Integral Equations of first, second and third kinds, Homogeneous Fredholm Integral Equation, Volterra integral equations of first, second and third kinds, Homogeneous Volterra integral equation, Singular Integral Equations.

Unit-II Symmetric Kernals, Degenerate Kernals, Iterated Kernals, Resolvent Kernals, Eigen values and Eigen functions of Integral operator, Solution by Eigen values and eigen functions Method.

Unit-III Solution of Fredholm Integral Equations of the second kinds with degenerate kernals, Method of Successive Approximations, Method of Successive Substitutions, Neumann series. Fredholm's First, second and third Fundamental Theorems, Green's Function. Modified Green's Function

Unit-IV Calculus of variations: Introduction, Euler - Lagrange equations, Invariance of Euler's Equations, Field of Extremals, Natural boundary conditions, Transversality conditions, Simple applications of variational principle, Sufficient conditions for extremum of a functional.

Unit-V Variational formulation of BVP, Moving Boundary problems, Euler's Finite Difference Method, Ritz Method, Variational methods for finding Eigenvalues and Eigenfunctions.

References:

[1] FB Hildebrand. *Methods of Applied Mathematics.* Prentice-Hall, 4th printing, 1958.

[2] Filip Rindler. *Calculus of variations.* Springer, 2018.

[3] AS Gupta. *Calculus of variations with applications.* PHI Learning Pvt. Ltd., 1996.

[4] MD Raisinghania. *Integral equations and boundary value problems.* S. Chand Publishing, 2007.

9.16 ME16: Fourier Analysis

Learning Outcomes: At the end of the course, the students will be able to :

1. Understand in-depth knowledge of Fourier analysis, discussion of convergence of Fourier series and its applications
2. Understand the summability of Fourier series for functions in L^1, L^2 and L^p spaces.
3. Calculate Fourier-transforms of integrable functions and basic properties of Fourier transforms.
4. Understand the concept of Distributions and Fourier Transforms, Tempered Distributions Applications to PDEs.

Contents:

Unit-I Fourier series, Discussion of convergence of Fourier series.

Unit-II Uniqueness of Fourier Series, Convolutions, Cesaro and Abel Summability, Fejer's theorem, Dirichlets theorem, Poisson Kernel and summability kernels. Example of a continuous function with divergent Fourier series.

Unit-III Summability of Fourier series for functions in L^1 , L^2 and L^p spaces. Fourier-transforms of integrable functions. Basic properties of Fourier transforms, Poisson summation formula, Hausdorff-Young inequality, Riesz-Thorin Interpolation theorem.

Unit-IV Schwartz class of rapidly decreasing functions, Fourier transforms of rapidly decreasing functions, Riemann Lebesgue lemma, Fourier Inversion Theorem, Fourier transforms of Gaussians, Plancheral theorem, Paley-Weiner theorem.

Unit-V Distributions and Fourier Transforms: Calculus of Distributions, Tempered Distributions: Fourier transforms of tempered distributions, Convolutions, Applications to PDEs.

References:

[1] Y Katznelson. An introduction to harmonic analysis, dover publications, new york, 1976.

[2] Robert E Edwards. *Fourier Series: A Modern Introduction Volume 2.* Springer Science & Business Media, 2012.

[3] Elias M Stein. *Fourier Analysis. An introduction.* 2003.

[4] Walter Rudin. *Fourier analysis on groups.* Courier Dover Publications, 2017.