School of Studies in Mathematics,
Pt.Ravishankar Shukla University Raipur

Syllabus

M.Sc. Mathematics Entrance Examination
(Semester System)

Admission Session: 2024-25 & onwards

CALCULUS:

$\varepsilon - \delta$ definition of the limit of a function. Basic properties of limits. Continuous functions and classification of discontinuties. Differentiability. Successive differentiation. Leibnitz theorem. Maclaurin and Taylor series expansions. Integration of transcendental functions. Reduction formulae. Definite integrals. Quadrature. Rectification. Volumes and surfaces of solids of revolution.

Definition of a sequence. Theorems on limits of sequences. Bounded and monotonic sequences. Cauchy's convergence criterion. Series of non-negative terms. Comparison tests, Cauchy's integral test, Ratio tests, Raabe's, logarithmic, de Morgan and Bertrand's tests. Alternating series, Leibnitz's theorem. Absolute and conditional convergence. Continuity, Sequential continuity, Properties of continuous functions, Uniform continuity, Chain rule of differentiability, Mean value theorems and their geometrical interpretations. Darboux's intermediate value theorem for derivatives, Taylor's theorem with various forms of remainders. Limit and continuity of functions of two variables. Partial differentiation. Change of variables. Euler's theorem on homogeneous functions. Taylor's theorem for functions of two variables. Jacobians. Envelopes, evolutes. Maxima, minima and saddle points of functions of two variables. Lagrange's multiplier method.

ALGEBRA:

Elementary operations on matrices, Inverse of a matrix. Linear independence of row and column matrices, Row rank, column rank and rank of a matrix. Equivalence of column and row ranks. Eigenvalues, eigenvectors and the characteristic equations of a matrix. Cayley Hamilton theorem and its use in finding inverse of a matrix. Application of matrices to a system of linear (both homogeneous and nonhomogeneous) equations. Theorems on consistency of a system of linear equations.

Mappings, Equivalence relations and partitions. Congruence modulo n. Definition of a group with examples and simple properties. Subgroups, generation of groups, cyclic groups, coset decomposition, Lagrange's theorem and its consequences. Fermat's and Euler's theorems. Normal subgroups. Quotient group, Permutation groups. Even and odd permutations. The alternating groups An. Cayley's theorem. Homomorphism and Isomorphism the fundamental theorems of homomorphism. Automorphisms, inner automorphism. Automorphism groups and their computations, Conjugacy relation, Normaliser, Counting principle and the class equation of a finite group. Center for Group of prime-order, Abelianizing of a group and its universal property. Sylow's theorems, Sylow subgroup, Structure theorem for finite Abelian groups. Ring theory-Ring homomorphism. Ideals and quotient rings. Field of quotients of an integral domain, Euclidean rings, polynomial rings, Polynomials over the rational field. The Eisenstien criterion, polynomial rings over commutative rings, Unique factorization domain. R unique factorisation domain implies so is R[x1, x2 ..... xn]. Modules, Submodules, Quotient modules, Homomorphism and Isomorphism theorems. Definition and examples of vector spaces. Subspaces. Sum and direct sum of subspaces. Linear span, linear dependence, independence and their basic properties. Basis. Finite dimensional vector spaces. Existence theorem for bases. Invariance of the number of elements of a basis set. Dimension. Existence of complementary subspace of a subspace of a finite dimensional vector space. Dimension of sums of subspaces. Quotient space and its dimension.

Linear transformations and their representation as matrices. The Algebra of linear transformations. The rank nullity theorem. Change of basis. Dual space. Bidual space and natural isomorphism. Adjoint of a linear transformation. Eigenvalues and eigenvectors of a linear transformation. Diagonalisation. Annihilator of a subspace. Bilinear, Quadratic and Hermitian forms. Inner Product Spaces-Cauchy-Schwarz inequality. Orthogonal vectors. Orthogonal Complements. Orthonormal sets and bases. Bessel's inequality for finite dimensional spaces. Gram-Schmidt Orthogonalization process.

DIFFERENTIAL EQUATIONS:

Degree an order of a differential equation. Equations reducible to the linear form. Exact differential equations. First order higher degree equations solvable for x, y, p. Clairaut's form and singular solutions. Geometrical meaning of a differential equation. Orthogonal trajectories. Linear differential equations with constant coefficients. Homogeneous linear ordinary differential equations. Linear differential equations of second order. Transformation of the equation by changing the dependent variable/the independent variable. Method of variation of parameters. Ordinary simultaneous differential equations.

Laplace Transformation- Linearity of the Laplace transformation, Existence theorem for Laplace transforms, Laplace transforms of derivatives and integrals, Shifting theorems. Differentiation and integration of transforms. Convolution theorem. Solution of integral equations and systems of differential equations using the Laplace transformation. Partial differential equations of the first order. Lagrange's solution, some special types of equations which can be solved easily by methods other than the general method, Charpit's general method of solution. Partial differential equations of second and higher orders, Classification of linear partial differential equations of second order, Homogeneous and non-homogeneous equations with constant coefficients, Partial differential equations reducible to equations with constant coefficients, Monge's methods.

ANALYSIS:

Series of arbitrary terms. Convergence, divergence and oscillation. Abel's and Dirichlet's test. Multiplication of series. Double series. Partial derivation and differentiability of real-valued functions of two variables. Schwarz and Young's theorem. Implicit function theorem. Fourier series. Fourier expansion of piecewise monotonic functions. Riemann integral. Intergrability of continuous and monotonic functions. The fundamental theorem of integral calculus. Mean value theorems of integral calculus. Improper integrals and their convergence. Comparison tests. Abel's and Dirichlet' tests. Frullani's integral. Integral as a function of a parameter. Continuity, derivability and integrability of an integral of a function of a parameter.

Complex numbers as ordered pairs. Geometric representation of complex numbers. Stereographic projection. Continuity and differentiability of complex functions. Analytic functions. Cauchy-Riemann equations. Harmonic functions. Elementary functions. Mapping by elementary functions. Mobius transformations. Fixed points, Cross ratio. Inverse points and critical mappings. Conformal mappings.

METRIC SPACES:

Definition and examples of metric spaces. Neighbourhoods, limit points, interior points, open and closed sets, closure and interior. Boundary points, Sub-space of a metric space. Cauchy sequences, completeness, Cantor's intersection theorem. Contraction principle, construction of real numbers as the completion of the incomplete metric space of rationals. Real numbers as a complete ordered field. Dense subsets. Baire Category theorem. Separable, second countable and first countable spaces. Continuous functions. Extension theorem. Uniform continuity, isometry and homeomorphism. Equivalent metrics. Compactness, sequential compactness. Totally bounded spaces. Finite intersection property. Continuous functions and compact sets, connectedness, components, continuous functions and connected sets.